Measuring What Matters in Value-Added Trade

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Abstract

The spread of global value chains (GVCs) has given rise to new statistical tools, the Inter-Country Input-Output tables, and new analytical frameworks aimed at properly identifying production linkages between and within economies. However, several important questions remain unaddressed. This paper proposes a new toolkit for value-added accounting of trade flows at the aggregate, bilateral, and sectoral levels. The paper shows how different empirical issues require distinct accounting perspectives and maps these methodologies onto the economic questions they are best suited to address. We provide novel accounting perspectives that allow us to properly address important empirical issues. With respect to other accounting methodologies previously proposed in the literature, we offer more accurate or, in some cases, more exhaustive value-added decompositions of trade flows (e.g., by covering both domestic and foreign value-added). In addition, the paper gathers a significant amount of the related literature under one comprehensive framework.

Keywords: trade in value-added; global value chains; inter-country input-output tables.
JEL classification: E16, F1, F14, F15.

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1 Introduction

The international fragmentation of production processes has challenged the capability of standard trade statistics to truly represent supply and demand linkages between economies. Whenever production is organized in sequential processing stages in different countries, trade statistics repeatedly double-count the same units of value-added, since they are shipped multiple times. The diffusion of global value chains (GVCs) has therefore deepened the divergence between gross flows, as recorded by traditional trade statistics, and the data on production and final demand as accounted for in statistics based on value-added (above all gross domestic product).

This paper develops a new toolkit of methodologies for value-added accounting of trade flows at the country, bilateral, and sectoral levels by using Inter-Country Input-Output (ICIO) tables (for instance WIOD, EORA and OECD/WTO TiVA; see Timmer et al., 2015; Lenzen et al., 2013, and Guilhoto et al., 2022). Then, it provides novel contributions to the literature that deals with the methodologies to define âdouble counted' components in order to identify the net âvalue-added' portion of trade flows (Koopman et al., 2014; Wang et al., 2013; Borin and Mancini, 2015, 2017; Los et al., 2016; Nagengast and Stehrer, 2016; Johnson, 2018; Los and Timmer, 2020; Miroudot and Ye, 2020). We suggest that the definition of double counting changes depending on the specific phenomenon under investigation. This gives rise to what we call alternative 'perspectives,' which are based on a common logical framework, each conceptually suited to addressing different types of empirical issues. For each accounting perspective, we trace the domestic and foreign value-added in trade flows. Our contribution to the literature is threefold: i) we extend the set of possible measures to address a wider range of empirical issues, ii) we improve the accuracy and comprehensiveness of some of the existing measures, and iii) we reconcile previous contributions under a unique comprehensive scheme.

In our framework, the perspective defines the perimeter according to which something is classified as 'value-added' or 'double counted,' the latter being the units of value-added that cross this perimeter more than once. The perimeter may consist of the entire array of collective international frontiers ('world perspective'), the national borders of one country ('exporter perspective'), the bilateral frontier between two countries ('bilateral perspective'), and one specific exporting sector of a given country ('sectoral perspective'). For instance, according to the 'world perspective,' a unit of value-added will be double counted if it is first exported from Germany to China and then re-exported from China to the United States. According to the 'exporter perspective,' double counting arises only when the same portion of value-added is exported twice by the same exporting country (e.g., Germany exports, re-imports, and eventually re-exports the same unit of value-added). By the same token, in the 'bilateral perspective,' the re-export should occur between the same two countries in order to be considered as double counted (e.g., Germany should export twice the same item to China). Following the same logic, we propose novel accounting perspectives (i.e., sectoral, sectoral-bilateral, importer, sectoral-importer) that are useful for addressing specific issues; for instance, we derive an analytical formula to compute the gross domestic product (GDP) of a country embedded in the total import of another country or in the import in a specific sector.

The perimeter according to which something is double-counted (i.e., the perspective) may or may not coincide with the level of disaggregation of the considered trade flow. The choice of the accounting perspective depends on the purpose of the analysis. For example, it is possible to use an 'exporter perspective' for value-added accounting of bilateral (or sectoral) trade flows. Indeed, we show that this is the correct choice when the analysis aims to break down the total value-added of a country by export market or sector of export. Conversely, measuring the value-added of countries exposed to a specific trade policy (e.g., an export quota or import tariff) may require a different accounting perspective, even when considering the same bilateral (or sectoral) trade flows. In the next section, we propose some examples of how empirical issues can be mapped on specific accounting perspectives.

Compared to the previous literature, we extend the bilateral accounting schemes of Johnson (2018) and Los and Timmer (2020) in two main directions: i) we generalize their approach to any level of aggregation of trade flows and to a wide range of accounting perspectives; ii) in each decomposition, we always consider both the value-added produced by the domestic country and by other countries (the so-called 'foreign value-added'). The latter component is key to assessing indirect linkages and spillovers through supply chains, but in previous contributions, it is either not considered or singled out using an accounting approach that differs from the one employed for the domestic component. For instance, Los and Timmer (2020) correctly analyzed bilateral trade flows using two accounting methods that correspond to the 'exporter perspective' and the 'bilateral perspective' in our framework; however, they focused only on domestic value-added.[1] Instead, Koopman et al. (2014) and Wang et al. (2013) used a 'world perspective' to single out the foreign value-added in total and bilateral exports, while the domestic part follows an 'exporter perspective.' This implies that the two indicators are not directly commensurable.

In general, vis-à-vis the existing literature, we demonstrate how the accounting methods presented in other studies can be associated with specific perspectives. Some contributions, such as those of Los et al. (2016), Johnson (2018), Los and Timmer (2020), and Miroudot and Ye (2020), are entirely consistent with the proposed framework. Other value-added indicators such as the 'import-content of exports' (VS) proposed by Hummels et al. (2001) and the (VAX) index by Johnson and Noguera (2012) correspond to specific subcomponents of the exporting country-perspective breakdowns. Other studies (e.g., Koopman et al. 2014; Wang et al. 2013; Nagengast and Stehrer 2016), while providing valuable contributions to the literature, are only loosely related to a specific perspective, since some components of these methodologies are imprecisely specified or inconsistent with the specific accounting approach. Some of these issues are discussed in greater detail in the following sections. However, this paper's contribution extends well beyond a critical review and systematic reorganization of existing methodologies, since most of the measures we propose do not find any correspondence in previous literature. Indeed, we consider novel 'perspectives' that allow us to address important empirical issues that have not been addressed yet.

Overall, the different accounting methods constitute a flexible toolkit that offers a broad scope for empirical investigations into global production networks at both the macro and micro levels. The framework proposed in this paper is implemented and ready-to-use in the Stata command *icio* (see Belotti, Borin and Mancini 2021). These indicators can be considered complementary to the broad set of measures included in the OECD TiVA database (oe.cd/tiva), which allows the decomposition of trade flows by country and sector of origin of the value-added without isolating the units of value-added that are double-counted in trade statistics (Guilhoto et al., 2022).

[1] A decomposition of bilateral trade flows based on an exporter perspective was originally proposed in Borin and Mancini (2017).

The remainder of this paper is organized as follows: Section 2 outlines our general framework, provides a mapping between accounting methods and empirical questions, and highlights the relationships with previous literature. Section 3 presents the methods that are best suited to compute the total GDP embedded in the exports of a country and its consistent breakdown across disaggregated trade flows. In particular, we apply it to aggregate exports (Section 3.1) and bilateral exports (3.2.1 and 3.2.2). Section 4 presents the accounting methods that are best suited to assess the amount of GDP that, given a certain trade flow, could be exposed to a particular shock, as a trade barrier; we consider bilateral exports (Section 4.1), sectoral-bilateral flows (4.2), sectoral exports (4.3), total aggregate imports (4.4), and sectoral imports (4.5). Section 5 provides relevant numerical examples. Section 6 concludes the paper.

2 Mapping different accounting perspectives with the empirical issues and the literature

The framework we propose consists of different methods, called 'perspectives,' to decompose trade flows. The perspective defines the perimeter according to which something is classified as 'value-added' or 'double counted,' the latter being the units of value-added that cross this perimeter more than once. A key issue is choosing the most appropriate accounting 'perspective' to address the empirical problem under investigation.

Generally, when the aim is to measure the *entire value-added of a country/sector of origin embedded in a given trade flow*, the accounting perspective to be applied should match the level of disaggregation of the trade flow considered. For instance, if country A imposes a tariff on the imports of textile and apparel from country B, and we are interested in the portion of country B value-added (or country C value-added) exposed to that trade barrier, we have to decompose this sectoral-bilateral trade flow (i.e., exports from A to B in the 'textile and apparel' sector) using a sectoral-bilateral perspective. In this case, the 'perimeter' that defines the double-counted units of value-added must coincide with the level of aggregation of the considered trade flow. It does not matter if the same item was previously exported by country A to another importing country or in another sector (e.g., as a raw material), since the value-added produced in previous production stages will be subject to the tariff as well. According to the sectoral bilateral accounting perspective, the only double-counted units of value-added are those exported more than once in the same sector to the same partner.

However, when we sum up indicators based on a narrower accounting perspective (e.g., bilateral or sector-bilateral), we obtain value-added measures that exceed the correct ones for aggregate trade flow (e.g., the total exports of a country). In other words, they are non-additive (Johnson, 2018; Los and Timmer, 2020). Thus, if we seek a *breakdown of the value-added measures by sectors/importing partners consistent with exporters' aggregate figures*, we need to apply the 'exporting country' perspective to the decomposition of more disaggregated trade flows. The resulting measures are additive. This additivity could be a desirable feature in some empirical applications (Nagengast and Stehrer, 2016, Xiao et al., 2020).[2]

[2] As we discuss in Section 3, this accounting perspective can also be used for other purposes such as to single out the portion of trade that is related to GVC activities (see Hummels et al. 2001, Borin and Mancini, 2015 and Antràs and Chor, 2022).

When the accounting perspective is set at a more aggregate level as compared to the considered trade flow (e.g., 'exporting country' perspective for bilateral exports), there is not a unique way to allocate value-added and double-counted units of value-added across the shipments. When the same component enters in various disaggregated trade flows we require economically meaningful criteria to decide in which case it has to be classified as 'value-added,' given that in the other case(s) it will be considered 'double counted.' To address this issue, two alternative approaches have been derived in the literature: the first method, the so called 'source-based' approach, accounts for the value-added the first time it leaves the country of origin; the second, 'sink-based' approach, considers it the last time it crosses the national borders. These approaches were conceived by Nagengast and Stehrer (2016) and were fully derived by Borin and Mancini (2015, 2017).

In addition, the choice between the two accounting methods depends on the particular empirical issue that we seek to address. In the source-based approach, the value added is recorded as closely as possible to the moment it is produced. It is then designed to examine the production linkages and country/sector participation in different types of production processes and to study the features of the production processes in which export flows are involved. For instance, drawing on this breakdown of gross trade, a set of indicators of GVC-related trade that consistently extend the 'Vertical Specialization' index (VS) proposed by Hummels et al. (2001) can be derived (see Borin and Mancini, 2015). Conversely, the value-added in the sink-based approach is recorded as closely as possible to the moment it is ultimately absorbed. Thus, it should be adopted to study the value-added composition of final goods' exports or to examine the role of a country's final demand in activating production and trade flows, such as in an analysis of bilateral trade balances (Borin and Mancini, 2016; Nagengast and Stehrer, 2016).

Using this general framework of accounting 'perspectives' and 'approaches,' it is possible to select the value-added measure of trade that is best suited for addressing the specific issue under investigation. In the following sections, we consider the measures that should be used to address the most common empirical questions. In particular, we show how domestic and foreign 'value-added' and domestic and foreign 'double counted' terms should be accounted for when the perimeter of interest is defined at the following levels: i) exporting country as a whole, ii) a bilateral relationship, iii) sectoral-bilateral relationship, iv) sector of export, v) importing country as a whole, and vi) sector of imports. In Appendix E, we also discuss the accounting of foreign value-added based on a 'world perspective' (i.e., when a given unit of value-added is accounted for as 'foreign value-added' only once in all possible trade flows).[3] For the exporting country perspective, we also show the difference between the source-based approach and the sink-based approach in the value-added decomposition of disaggregated trade flows. Table 1 provides a (non-exhaustive) overview of some common empirical questions and the related accounting methodology required to address them, and Section 5 provides numerical examples.

Table 2 shows how the measures proposed in this paper relate to the other original contributions in the literature. Some methodologies, such as Los et al. (2016) and Johnson (2018), found a perfect mapping in the proposed framework. The contribution

[3]We include the source and sink decompositions based on a 'world perspective' as derived in Borin and Mancini (2017) to have a complete mapping with other contributions in the literature that follow this approach (Koopman et al. 2014; Wang et al. 2013; Miroudot and Ye, 2020). However, we argue that the exports' decompositions based on this perspective do not provide any economically meaningful information in addition to what we obtain through the other perspectives. See Appendix E for further details.

Table 1: Overview of the most common empirical questions

Empirical question	**Trade flow to select**	**Perspective**
GDP embedded in the total exports of a country	total aggregate exports	exporting-country
GDP potentially exposed to:		
— a shock on a bilateral trade relation (e.g. generic trade frictions between two countries)	bilateral aggregate exports	bilateral
— a shock on a specific sectoral-bilateral trade relation (e.g. a specific tariff imposed by a trade partner in a given sector)	bilateral sectoral exports	sectoral-bilateral
— a shock on the imports of a country (e.g. trade restrictions vis-à-vis all partners)	total aggregate imports	importing-country
— a shock on the imports of a country in a given sector (e.g. a specific sectoral tariff vis-à-vis all partners)	total sectoral imports	sectoral-importer
— a shock on the sectoral exports of a country (e.g. negative demand shock on the exports of a given country and sector)	total sectoral exports	sectoral-exporter
Value-added breakdown in disaggregated export flows, consistent with total aggregate measures	sectoral/bilateral/sectoral-bilateral exports	exporting-country, source or sink
Value-added breakdown of bilateral trade balances	bilateral exports	exporting-country, sink
Traditional exports vs GVC-exports	any export flow	exporting-country, source

Table 2: Summary of the different measures of value-added in trade and relationships with original contributions in the literature

Perspective	**Trade flow decomposed**	**Approach**	**Origin of value-added**	**Original contribution**	**Paper section**
Exporting country	Total exports		Domestic Foreign	Koopman et al. (2014), Los et al. (2016) Borin & Mancini (2017)	Section 3.1
Exporting country	Bilateral exp. [a]	Source	Domestic Foreign	Nagengast & Stehrer (2016)[b], Borin & Mancini (2015) Borin & Mancini (2017)	Section 3.2.1
Exporting country	Bilateral exp.[a]	Sink	Domestic Foreign	Wang et al. (2013)[c], Borin & Mancini (2015) Borin & Mancini (2017)	Section 3.2.2
Bilateral	Bilateral exp.		Domestic Foreign	Los et al. (2016) Johnson (2018)[d], **new**	Section 4.1
Sectoral-bilateral	Sectoral-bilateral exp.		Domestic Foreign	**new** **new**	Section 4.2
Exporting sector	Sectoral exp.		Domestic Foreign	**new** **new**	Section 4.3
Importing country	Total imports		Country of origin	**new**	Section 4.4
Importing sector	Sectoral imports		Country of origin	**new**	Section 4.5
World	Total/bilateral exp.	Source Sink	Foreign Foreign	Borin & Mancini (2017) Koopman et al. (2014)[e], Borin & Mancini (2015)	Appendix E

(a) Correspondent decompositions for total exports can be obtained summing across importing countries. In this case, sink and source breakdowns provide the same results for DVA and FVA. (b) In Nagengast and Stehrer (2016) the distinction between sink and source is implemented only for a sub-portion of their decompositions (i.e. for the direct absorption of DVA by the bilateral partner). Moreover, even for this sub-portion, the sink decomposition is inaccurately specified. (c) Wang et al. (2013) propose a decomposition of bilateral trade flows based on the 'exporting country perspective' for the domestic content of exports. However, some sub-components of the decomposition are based on a sink approach, while others follow a source approach. (d) Johnson (2018) develops a decomposition based on a bilateral perspective both for domestic and foreign value-added, but only in a two-country setting. (e) Koopman et al. (2014) and Wang et al. (2013) underestimate the correct measure of foreign value-added in exports, see discussion in Appendix D and E.

of this paper–together with its previous unpublished versions (Borin and Mancini, 2015, 2017)–with respect to these works, consists in providing a more exhaustive coverage in terms of accounting perspectives and the origin of value-added. Other works in the literature introduced important innovations: for instance, Koopman et al. (2014) originally proposed a way to single out value-added in aggregate trade flows, separating it from double-counted items, Wang et al. (2013) extended the framework to bilateral exports, Nagengast and Stehrer (2016) proposed the idea of having different decompositions for bilateral trade flows (i.e., sink and source). However, these methodologies present some shortcomings and limitations that lead to imprecise or incorrect evaluations as compared with those presented in this paper and in other contributions that are entirely consistent with the proposed framework (e.g., Los et al. 2016; Johnson, 2018; Los and Timmer, 2020; Miroudot and Ye, 2020).

Not all existing accounting methods are conceptually correct or economically meaningful. Each method finds its theoretical justification for measuring a certain phenomenon properly and/or for addressing an economically relevant question. Therefore, every decomposition of trade flows should meet two basic requirements: i) appropriate labeling (i.e., each component should correctly identify what it is supposed to measure) and ii) internal consistency (i.e., each component should be consistent with the specific approach adopted). Some relevant contributions in the literature do not completely satisfy these criteria; therefore, they are not fully consistent with the accounting framework proposed in this paper. For instance, Koopman et al.'s (2014) classification does not properly allocate the domestic value-added in exports between the part eventually absorbed by direct importers and the part absorbed in third markets; in addition, a portion of the foreign content of exports is erroneously classified as 'double counted' whereas it should be allocated to the 'foreign value-added' according to the criterion chosen by the authors. The latter problem also affects Wang et al.'s (2013) decomposition of bilateral exports; moreover, they used different accounting approaches to single out the components of their decomposition, making it internally inconsistent. Finally, despite introducing important conceptual innovations, neither methodology proposed by Nagengast and Stehrer (2016) provides an accurate breakdown of the entire domestic and foreign value-added exported by a country across different bilateral flows. In the next sections, when presenting the different accounting perspectives, we also discuss the differences between these methodologies in a thorough and technical manner. Further details are reported in Appendices D and E.

3 Breaking down a country's value-added across trade flows: the exporting country perspective

The exporting country-level perspective is suited to measuring the GDP of a country embedded in its own total exports ('domestic value-added') or in the total exports of any other country ('foreign value-added'). Thus, this is the perspective that, in general, should be considered when analyzing aggregate export flows. This perspective may also be used to measure the value added in disaggregated trade flows (i.e., bilateral, sectoral, sectoral-bilateral) when we want to preserve the additivity of the indicators at the level of the exporting country.

The measures that we present are based on a standard Inter-Country Input-Output (ICIO) model with G countries and N sectors. Appendix A provides an exhaustive definition of the notation and, for this reason, here we only mention that $\mathbf{E}_{sr}$ is the $N\times 1$ vector of exports of country s to country r, $\mathbf{X}_s$ is the $N\times 1$ vector of gross output produced by country s, $\mathbf{A}$ is the $GN\times GN$ global matrix of input coefficients, $\mathbf{B}$ is the global Leontief inverse matrix for the entire inter-country model and $\mathbf{V}_s$ is the $1\times N$ vector that incorporates the value-added shares embedded in each unit of gross output produced by country s.

3.1 Exporting country perspective decomposition of aggregate exports

Drawing from Borin and Mancini (2017), we propose a breakdown of aggregate exports that is fully consistent with the exporting country perspective.

The total gross exports of country s ($\mathbf{E}_{s*}$) can be broken down according to the country that initially produced each component by using the global Leontief inverse matrix—which traces back the total gross output produced by each country j to deliver one unit of country s exports ($\mathbf{B}_{js}$)—and the related value-added shares ($\mathbf{V}_j$, see Koopman et al. 2010):

$$\mathbf{u}_N\mathbf{E}_{s*} = \underbrace{\mathbf{V}_s\mathbf{B}_{ss}\mathbf{E}_{s*}}_{\substack{\text{domestic}\\ \text{content } (\mathbf{DC}_s)}} + \underbrace{\sum_{t\neq s}^{G}\mathbf{V}_t\mathbf{B}_{ts}\mathbf{E}_{s*}}_{\substack{\text{foreign}\\ \text{content } (\mathbf{FC}_s)}}. \tag{1}$$

The accounting relationship in Equation (1) allows to break down gross exports by country of origin. Although these components cannot be directly related to the GDP of the country of origin as they include value-added units that are double-counted, these measures can be useful in several empirical applications. For instance, using various refined versions of Equation (1) to consider the industry dimension, the OECD-TiVA website (oe.cd/tiva) provides a very broad set of indicators on the origin of trade flows and final demand (see Guilhoto et al. 2022).[4] Nonetheless, the components in (1) are still 'gross accounting,' because they include the units of value-added that cross a country's borders several times during the production process. Only by isolating these double-counted components can we measure the 'net' production (value-added) embedded in exports, akin to countries' GDP.

According to the exporting country perspective, double counting in the total gross exports of a given country s occurs whenever certain components that are first exported by s are then re-imported and used to produce goods and services to be exported again. Conceptually, one possible way to distinguish between 'value-added' and 'double counting' is to split the production chain in phases, each delimited by the export flow of country s: what is generated within that particular production phase is accounted for as 'value-added' in exports, what comes from further upstream production stages is 'double counted.'

For instance, in the basic production process depicted in Figure 1, the value produced in 'phase 1' should be accounted for as 'value-added' in the first shipment from s, whereas in the last shipment it should be considered as 'double counted'. Indeed, in the

[4]It is important to highlight that in the TiVA database the different components are labeled 'domestic (foreign) value-added content.' Conversely, following Koopman et al. (2014), we refer to the same components as 'domestic (foreign) content,' while the 'domestic (foreign) value-added' in our wording indicates the 'net' part of the 'domestic/foreign content' (i.e., the units of value-added that are not double counted). See also the decomposition scheme by Koopman et al. (2014) in Appendix D.

decomposition of 'shipment 2' from s, only the value produced by s and r in 'phase 2' is accounted for as 'value-added'. In other words, starting from 'shipment 2', we should go back up the production chain to 'shipment 1' (i.e. up to the point in which country r imports intermediate inputs from country s).

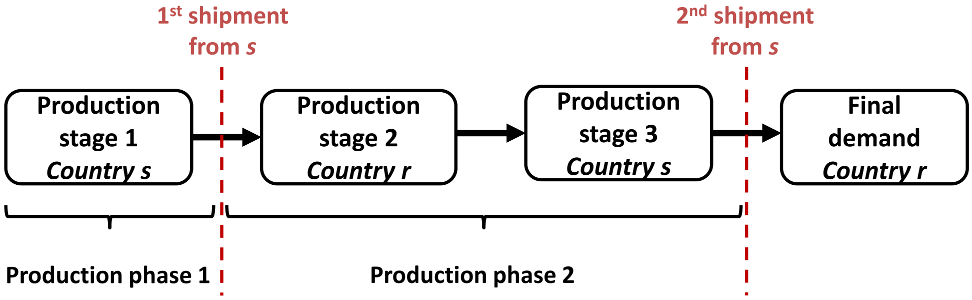

Figure 1: Breakdown of production phases

This partition of the production process can be implemented in a general ICIO framework by modifying matrix $\mathbf{B}$ in such a way that we can slice down the production process along the outward boundaries of the exporting country s. To this end, consider the representation of the global Leontief inverse as a sum of infinite series of the gross output generated in all upstream stages of the production process:

$$\mathbf{B} = \mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \mathbf{A}^3 + \ldots + \mathbf{A}^n \quad n \to \infty. \tag{2}$$

We can split the production process along country s's borders by carving out its intermediate export linkages at any stage of the above series. Algebraically, it can be implemented by setting the coefficients of matrix $\mathbf{A}$ that identify the direct requirement of intermediate inputs from country s to zero. Then, we can define a new input requirement matrix $\mathbf{A}^{\not s}$ such that $\mathbf{A}^{\not s}_{sj} = 0 \ \forall \ j \neq s$, while $\mathbf{A}^{\not s}_{ij} = \mathbf{A}_{ij}$ if $i \neq s$ or $i = j = s$. The corresponding inverse Leontief matrix $\mathbf{B}^{\not s}$ is simply equal to $(\mathbf{I} - \mathbf{A}^{\not s})^{-1}$ and the following relationship holds true (see Appendix B):

$$\mathbf{B}_{is} = \mathbf{B}^{\not s}_{is} + \mathbf{B}^{\not s}_{is} \sum_{j \neq s} \mathbf{A}_{sj} \mathbf{B}_{js}, \tag{3}$$

where i could be either s or a different country.

Using (3), equation (1) can be rewritten to single out the "value-added" and "double counted" terms within each component:

$$\mathbf{u}_N \mathbf{E}_{s*} = \overbrace{\underbrace{\mathbf{V}_s \mathbf{B}^{\not s}_{ss} \mathbf{E}_{s*}}_{\substack{\text{domestic value}\\\text{added}\\(\mathbf{DVA}_{s*})}} + \underbrace{\mathbf{V}_s \mathbf{B}^{\not s}_{ss} \sum_{j \neq s} \mathbf{A}_{sj} \mathbf{B}_{js} \mathbf{E}_{s*}}_{\substack{\text{domestic double}\\\text{counted}\\(\mathbf{DDC}_{s*})}}}^{\substack{\text{domestic}\\\text{content } (\mathbf{DC}_{s*})}} + \overbrace{\underbrace{\sum_{t \neq s}^{G} \mathbf{V}_t \mathbf{B}^{\not s}_{ts} \mathbf{E}_{s*}}_{\substack{\text{foreign value}\\\text{added}\\(\mathbf{FVA}_{s*})}} + \underbrace{\sum_{t \neq s}^{G} \mathbf{V}_t \mathbf{B}^{\not s}_{ts} \sum_{j \neq s} \mathbf{A}_{sj} \mathbf{B}_{js} \mathbf{E}_{s*}}_{\substack{\text{foreign double}\\\text{counted}\\(\mathbf{FDC}_{s*})}}}^{\substack{\text{foreign}\\\text{content } (\mathbf{FC}_{s*})}} . \tag{4}$$

Equation (4) reproduces the breakdown of aggregate exports into the main items identified in Koopman et al. (2014). The double-counted components are measured by isolating the portion of country s exports that have already been exported by s in a previous stage of the production process. With regard to domestic components, it is worth noting that $\mathbf{B}^{\nexists}_{ss}$ corresponds to the so-called local Leontief matrix $(\mathbf{I}-\mathbf{A}_{ss})^{-1}$. This means that the domestic value-added in exports is obtained by isolating all the domestic stages of production needed to produce exported goods while ignoring the domestic content of imported inputs. Notably, this measure of domestic value-added in exports represents the complement to the 'import content of exports' proposed by Hummels et al. (2001) (see proof in Appendix C), but it is also numerically equivalent to the domestic value added in other contributions that have analyzed aggregate export flows (e.g., Koopman et al., 2014; Los et al., 2016; Johnson, 2018).

The foreign value-added in (4) follows the same rationale, i.e. considering as value-added only the items crossing country s border once. If we focus on a specific country of origin t, the foreign value-added component $\mathbf{V}_t\mathbf{B}^{\nexists}_{ts}\mathbf{E}_{s*}$ that appears in equation (4) measures the entire GDP produced by t and exported by s. However, this is not the indicator of "foreign value-added" adopted by other contributions in the literature (See Appendix E).[5]

The components in equation (4) can be further broken down according to the country of ultimate absorption in the final demand. Indeed, each export flow from country s to country r ($\mathbf{E}_{sr}$) consists of final goods ($\mathbf{Y}_{sr}$) and intermediate inputs for the production of gross output in country r ($\mathbf{X}_r$):

$$\mathbf{E}_{sr} = \mathbf{Y}_{sr} + \mathbf{A}_{sr}\mathbf{X}_r, \tag{5}$$

where $\mathbf{Y}_{sr}$ indicates the $N \times 1$ vector of final goods completed in s and consumed in r. The intermediate inputs imported by country r ($\mathbf{A}_{sr}\mathbf{X}_r$) can be linked to the country of final completion and the final destination market by the Leontief inverse matrix $\mathbf{B}$, which considers the remaining production stages (see equation A.2 in Appendix A). Then, the aggregate exports of country s are linked to the final market of destination l by the following relationship:

$$\mathbf{E}_{s*} = \sum_{l\neq s}^{G}\mathbf{Y}_{sl} + \sum_{r\neq s}^{G}\mathbf{A}_{sr}\sum_{k}^{G}\sum_{l}^{G}\mathbf{B}_{rk}\mathbf{Y}_{kl}, \tag{6}$$

By substituting the total exports in Equation (4) with the relationship in Equation (6), we obtain a complete decomposition of aggregate exports by origin and destination. For instance, the domestic value-added ultimately absorbed in the country of origin itself, i.e. the 'reflection' terms in Koompan et al. (2014) terminology, can be singled out considering only the case in which the market of final demand l corresponds to s in the

[5] Other methodologies which also use a country-level perspective for the domestic componentâ Koopman et al. (2014), Wang et al. (2013), Nagengast and Stehrer (2016), Miroudot and Ye (2020)âtake a different approach for the foreign content of exports: a certain item is considered value-added only the first (or the last) time it crosses a foreign border, whereas all the other times it crosses any foreign border it is classified as double counted. We can label this approach a 'world-level perspective,' since all trade flows—not only the exports of a single country—are considered to single out the units of value-added that are exported multiple times. In other words, with the 'country-level perspective,' a certain unit of value-added is accounted for as foreign value-added only once in the total exports of a country, whereas the 'world-level perspective' requires it to be accounted for as foreign value-added only once in total world exports. See Appendix E for a thorough discussion.

second term on the right-hand side of (6). Conversely, by setting $l \neq s$ we were able to obtain the domestic value-added absorbed in a foreign market, which corresponds to the 'value-added exports,' or $\mathbf{VAX}_{s*}$, in Johnson and Noguera (2012) nomenclature.

Some empirical questions which can be addressed with this perspective are:

- What is the GDP embedded in the total exports of a country?
- What is the foreign GDP embedded in the total exports of a country?
- What is the GDP embedded in the total exports of a country re-imported and consumed at home?

3.2 Exporting country perspective decomposition of bilateral exports

The exporting country perspective can be applied to the decomposition of disaggregated trade flows to preserve the additivity of the indicators. As mentioned above, Nagengast and Stehrer (2016) pointed out that this type of decomposition may follow either a source-based approach that is more supply oriented or a sink-based approach that is more demand-oriented. Here, we present two decompositions according to the formulations developed by Borin and Mancini (2015, 2017) which are both accurate and internally consistent. Then, we briefly discuss the main differences compared to the accounting methods proposed by Nagengast and Stehrer (2016) and Wang et al. (2013), who also proposed a decomposition of bilateral exports mainly based on an exporting country perspective.

3.2.1 Bilateral source-based breakdown

In the source-based breakdown of bilateral exports, the value produced in a country is accounted for as 'value-added' the first time it crosses the exporter's national borders. Indeed, the specific approach that we adopted to derive value-added accounting of aggregate exports (i.e., accounting for as 'value-added' what is generated within that particular production phase and as 'double counted' what comes from further upstream production stages) meets this requirement. Then, we can apply the algebraic device presented in Section 3.1, i.e. the $\mathbf{B}^{\not s}$ matrix, to single out value-added components in bilateral flows according to an exporter-perspective source-based approach. In particular, the decomposition is obtained by substituting the bilateral export flow $\mathbf{E}_{sr}$ to the aggregate exports $\mathbf{E}_{sr}$ in equation (4):

$$\mathbf{u}_N\mathbf{E}_{sr} = \overbrace{\underbrace{\mathbf{V}_s\mathbf{B}^{\not s}_{ss}\mathbf{E}_{sr}}_{\substack{\text{domestic value}\\ \text{added}\\ (\mathbf{DVA}source_{sr})}} + \underbrace{\mathbf{V}_s\mathbf{B}^{\not s}_{ss}\sum_{j\neq s}\mathbf{A}_{sj}\mathbf{B}_{js}\mathbf{E}_{sr}}_{\substack{\text{domestic double}\\ \text{counted}\\ (\mathbf{DDC}source_{sr})}}}^{\substack{\text{domestic}\\ \text{content } (\mathbf{DC}_{sr})}} + \overbrace{\underbrace{\sum_{t\neq s}^{G}\mathbf{V}_t\mathbf{B}^{\not s}_{ts}\mathbf{E}_{sr}}_{\substack{\text{foreign value}\\ \text{added}\\ (\mathbf{FVA}source_{sr})}} + \underbrace{\sum_{t\neq s}^{G}\mathbf{V}_t\mathbf{B}^{\not s}_{ts}\sum_{j\neq s}\mathbf{A}_{sj}\mathbf{B}_{js}\mathbf{E}_{sr}}_{\substack{\text{foreign double}\\ \text{counted}\\ (\mathbf{FDC}source_{sr})}}}^{\substack{\text{foreign}\\ \text{content } (\mathbf{FC}_{sr})}}. \tag{7}$$

Equation (7) reports a source-based breakdown of bilateral exports by the country of origin. The double-counted items are measured by isolating the portion of country s'

exports to r that has already been exported by s in a previous stage of the production process.[6]

As already mentioned, it is worth noting that $\mathbf{B}^{\not s}_{ss}$ corresponds to the local Leontief matrix $(\mathbf{I}-\mathbf{A}_{ss})^{-1}$ (see equation (D.9) in Appendix D). This indicates the domestic value-added in exports ($\mathbf{DVA}source_{sr}$) in the source-based approach is obtained by isolating all the domestic stages of production required to produce the exported goods, while ignoring the domestic content of imported inputs ($\mathbf{V}_s(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{E}_{sr}$). Notably, this measure of domestic value-added in exports corresponds to that proposed by Johnson (2018), but only in the two-country world that the author considers. In a more general framework with several countries and sectors, Johnson's (2018) approach appears different from the exporting country-level source-based approach presented here.[7]

In addition to the breakdown of value-added by country of origin, the literature has also considered the relationship with the market of final absorption (Johnson and Noguera, 2012, Koopman et al., 2014). In a bilateral context, we can dig deeper into forward production linkages and connections with final demand. Since infinite rounds of production could occur before an intermediate product reaches final demand, we stress that our choice is to identify the direct importer, (potential) second destination of re-export, country of completion of final products, and ultimate destination market.

As shown in (5), we can split bilateral exports $\mathbf{E}_{sr}$ into final goods ($\mathbf{Y}_{sr}$) and intermediate inputs required by the production of gross output of country r ($\mathbf{A}_{sr}\mathbf{X}_r$). In country r, the intermediate inputs imported from s undergo one or more processing phases to produce final products for domestic consumption or goods for re-export (both intermediate and final):

$$\mathbf{E}_{sr} = \mathbf{Y}_{sr} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{Y}_{rr} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{E}_{r*}. \tag{8}$$

By applying equation (6) to the (re)exports from country r ($\mathbf{E}_{r*}$), we obtain the decomposition of bilateral exports by the market of final destination:

$$\mathbf{E}_{sr} = \mathbf{Y}_{sr} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{Y}_{rr} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\left[\mathbf{Y}_{rj} + \mathbf{A}_{rj}\sum_{k}^{G}\sum_{l}^{G}\mathbf{B}_{jk}\mathbf{Y}_{kl}\right]. \tag{9}$$

As discussed for the aggregate exports, by combining equations (7) and (9), we can obtain a comprehensive source-based decomposition of bilateral exports both by origin and final destination. See Appendix G for the complete expressions of the main components.

By exploiting all the information in Equations (7) and (9), it is possible to isolate any specific market of final absorption (r, j and l), country of re-export (j) and country of final completion (s, r and k). For instance, in this way we can single out the value-added generated in s and absorbed directly by the importer country r without any further re-export, i.e. the 'directly absorbed value-added in exports', or $\mathbf{DAVAX}_{sr}$):

$$\mathbf{DAVAX}_{sr} = \mathbf{V}_s(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{Y}_{sr} + \mathbf{V}_s(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{Y}_{rr}. \tag{10}$$

[6] Part of the intermediate goods exported by country s ($\sum_{j\neq s}^{G}\mathbf{A}_{sj}\mathbf{X}_j$) are later re-imported by s itself and enter again its exports, generating in this sense a double-counted item in a source-based framework. In particular, we are interested in the intermediate goods shipped abroad that re-enter in the exports from s to r, following any possible production pattern ($\sum_{j\neq s}^{G}\mathbf{A}_{sj}\mathbf{X}_j^{(\to\mathbf{E}_{sr})}$). This can be computed as: $\sum_{j\neq s}^{G}\mathbf{A}_{sj}\mathbf{B}_{js}\mathbf{E}_{sr}$.

[7] We discuss this point more extensively in Section 4.1, where we propose an alternative decomposition of gross exports that generalizes to an n-country context the approach suggested by Johnson (2018).

The $\mathbf{DAVAX}_{sr}$ identifies the 'traditional' type of trade/production as opposed to the international shipments that occur under the global sharing of production ('GVC-related trade'). As highlighted in Borin and Mancini (2015), 'GVC-related trade' includes all the traded value-added components that cross at least two international borders, that is, they are re-exported at least once before being absorbed in final demand. This can be considered a sufficient condition for an exported good to be part of an international production network and, at the aggregate level, it represents 'a natural measure of the importance of GVC trade in total international trade' (Antràs, 2020; Antràs and Chor, 2022).

The $\mathbf{DAVAX}_{sr}$ indicator can be computed only in a source-based approach and differs from the sum of the first two terms in Koopman et al.'s (2014) decomposition, even when summing across all bilateral destinations. Indeed, although Koopman et al. (2014) refer to these terms as 'the domestic value-added in intermediate exports absorbed by direct importers,' these subcomponents are not correctly identified in their decomposition. More specifically, they calculate them as $\mathbf{V}_s \sum_{r \neq s}^{G} \mathbf{B}_{sr} \mathbf{Y}_{rr}$; they also claim that the 'domestic value-added in intermediate goods re-exported to third countries' is given by $\mathbf{V}_s \sum_{r \neq s}^{G} \sum_{t \neq s,r}^{G} \mathbf{B}_{sr} \mathbf{Y}_{rt}$. In fact, both expressions refer to the domestic value-added absorbed in any foreign market; the difference between the two components is that, in the first case, final goods are completed and consumed in the same country, whereas in the latter, goods are completed in a foreign country and absorbed in a different country. The problem is that the global inverse Leontief matrix $\mathbf{B}_{sr}$ employed by Koopman et al. (2014) does not trace a bilateral exporter-importer linkage.

Similarly, also Wang et al. (2013) decomposition of bilateral export is not suited to single out the **DAVAX** indicator for two reasons: *i*- the DVA in exports of final goods is measured using the global inverse Leontief matrix as in Koopman et al. (2014) (i.e. $\mathbf{V}_s \mathbf{B}_{ss} \mathbf{Y}_{sr}$); *ii*- it does not single out the intermediate goods absorbed by the bilateral importer without additional processing stages abroad. Indeed, in a revised version of their work (Wang et al. 2018), the authors acknowledge this limitation and refer to the source-based decomposition by Borin and Mancini (2015) presented above to measure GVC-related trade.

Borin et al. (2021) provide a comprehensive description of GVC participation measures based on the **DAVAX** indicator. In general, some empirical questions that can be addressed using the source-based exporter-perspective accounting are the following:

- What is the domestic value-added absorbed directly by the bilateral partner?
- What is the global value-chain related exports of a country?
- What is the domestic value-added in bilateral flows traced the first time it crosses the national border, consistent with total export flows?
- What is the foreign value-added in bilateral flows traced the first time it crosses the national border, consistent with total export flows?

3.2.2 Bilateral sink-based breakdown

While the source-based approach discussed above is most suited to examining production linkages, the sink-based approach is most appropriate when the focus is on final demand and how it relates to the total value-added produced in a country. This is because in the sink-based decomposition, a given item is accounted for as 'value-added' the last time it leaves national borders, and, in the case of multiple crossing, it is considered 'double counted' in prior shipments. Reconsidering, for instance, the illustrative example in Figure 1, the whole value-added generated in phases 1 and 2 is accounted for as such in the last shipment from country s (i.e. in shipment 2), whereas the value of shipment 1 is entirely attributed to the double-counted term. Then, to single out the 'value-added' components in a sink-based framework, it is necessary to isolate the portion of ultimate shipments within a certain bilateral trade flow. These 'ultimate exports' ($\mathbf{E}_{sr}^{(\nrightarrow \mathbf{Y}_*)}$) are comprised of final goods ($\mathbf{Y}_{sr}$) and of intermediate goods that do not re-enter country s exports, before reaching the ultimate destination ($\mathbf{A}_{sr}\mathbf{X}_j^{(\nrightarrow \mathbf{Y}_*)}$). Since the latter are commensurate with final goods as concerns the exporting country s, the overall value-added can be computed by pre-multiplying the vector of 'ultimate exports' by the **VB** matrix. In other words, once the part of 'ultimate exports' is singled out, the value-added in exports can be computed in the same way as the **VBY** matrix is used to measure the total value-added in final demand (see Appendix A). In particular, the global Leontief inverse matrix **B** takes into account all upstream production stages as required by the sink-based approach. Conceptually, assuming that we can split bilateral exports between ultimate shipments ($\mathbf{E}_{sr}^{(\nrightarrow \mathbf{Y}_*)}$) and exports of intermediates that will later be re-exported by s itself ($\mathbf{E}_{sr}^{(\to \mathbf{E}_{s*})}$), the essential value-added breakdown of bilateral exports in a sink-based framework can be expressed as follows:

$$\mathbf{u}_N\mathbf{E}_{sr} = \overbrace{\underbrace{\mathbf{V}_s\mathbf{B}_{ss}\mathbf{E}_{sr}^{(\nrightarrow \mathbf{Y}_*)}}_{\substack{\text{domestic value}\\ \text{added } (\mathbf{DVA}sink_{sr})}} + \underbrace{\mathbf{V}_s\mathbf{B}_{ss}\mathbf{E}_{sr}^{(\to \mathbf{E}_{s*})}}_{\substack{\text{domestic double}\\ \text{counted } (\mathbf{DDC}sink_{sr})}}}^{\substack{\text{domestic}\\ \text{content } (\mathbf{DC}_{sr})}} + \overbrace{\underbrace{\sum_{t\neq s}^{G}\mathbf{V}_t\mathbf{B}_{ts}\mathbf{E}_{sr}^{(\nrightarrow \mathbf{Y}_*)}}_{\substack{\text{foreign value}\\ \text{added } (\mathbf{FVA}sink_{sr})}} + \underbrace{\sum_{t\neq s}^{G}\mathbf{V}_t\mathbf{B}_{ts}\mathbf{E}_{sr}^{(\to \mathbf{E}_{s*})}}_{\substack{\text{foreign double}\\ \text{counted } (\mathbf{FDC}sink_{sr})}}}^{\substack{\text{foreign}\\ \text{content } (\mathbf{FC}_{sr})}}. \tag{11}$$

To make the breakdown in equation (11) operational, we must identify the 'ultimate shipping'. We then proceed by disentangling the bilateral flow $\mathbf{E}_{sr}$, as we did for the source-based approach, to identify the downstream linkages with final demand. By making use of the relations in equations (8)–(9), we can express bilateral exports as follows:

$$\mathbf{E}_{sr} = \mathbf{Y}_{sr} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{Y}_{rr} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{Y}_{rj} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{A}_{rj}\mathbf{X}_j. \tag{12}$$

The first three terms on the right-hand side of equation (12) are clearly part of the 'ultimate shipment' of country s, since the value-added reaches the final demand without any re-shipment from s. At this point, we need to define the part of country j's output that passes through country s's export borders ($\mathbf{X}_j^{(\to \mathbf{E}_{s*})}$) and the part that reaches the final demand without any re-shipment from s ($\mathbf{X}_j^{(\nrightarrow \mathbf{Y}_*)}$). The problem is very similar to the one we faced in the source-based decomposition, when we singled out the portion of exports that crossed country s's export border for the first time. In this case, the algebraic solution is found by recurring to the modified version of the Leontief inverse matrix that excludes the intermediate export linkages from country s ($\mathbf{B}^{\not s}$). In this way, we can take into account all the possible patterns through which country j's output reaches the final demand, with the exception of those that involve a re-export from s. By following a procedure similar to that used to derive equation (4), country j's gross production can be decomposed as:

$$\mathbf{X}_j = \underbrace{\sum_{k \neq s}^{G} \sum_{l}^{G} \mathbf{B}^{\not s}_{jk} \mathbf{Y}_{kl} + \mathbf{B}^{\not s}_{js} \mathbf{Y}_{ss}}_{\mathbf{X}_j^{(\overset{\not s}{\rightarrow} \mathbf{Y}_*)}} + \underbrace{\mathbf{B}^{\not s}_{js} \mathbf{E}_{s*}}_{\mathbf{X}_j^{(\rightarrow \mathbf{E}_{s*})}} . \tag{13}$$

The decomposition in equation (13) allows us to identify the part of country j's production that is not part of country s's exports before reaching the ultimate destination ($\mathbf{X}_j^{(\overset{\not s}{\rightarrow} \mathbf{Y}_*)}$), whereas the remaining component identifies the double-counted terms ($\mathbf{X}_j^{(\rightarrow \mathbf{E}_{s*})}$). By combining equations (11), (12) and (13) we obtain the main terms of the sink-based breakdown of bilateral exports (i.e. the domestic value-added, $\mathbf{DVA}sink_{sr}$, the foreign value-added, $\mathbf{FVA}sink_{sr}$, the domestic double counted, $\mathbf{DDC}sink_{sr}$, and the foreign double counted, $\mathbf{FDC}sink_{sr}$). See Appendix G for a detailed expression of each component.

As in the case of source-based accounting, the different components of sink-based decomposition can be used to isolate the value-added in exports for one or more of the highlighted actors (i.e., exporter, importer, origin of the value-added, market of re-export, country of final completion, and ultimate absorption). For instance, in this case, we can also distinguish between domestic value-added that is finally absorbed at home ($\mathbf{REF}sink_{sr}$) or abroad ($\mathbf{VAX}sink_{sr}$) by selecting the proper vector of final demand ($\mathbf{Y}_{jl}$).

A subcomponent of the $\mathbf{VAX}sink_{sr}$ that may be economically interesting is the domestic value-added that is finally absorbed by the importer country r itself:

$$\mathbf{VAXIM}_{sr} = \mathbf{V}_s \mathbf{B}_{ss} \left[\mathbf{Y}_{sr} + \mathbf{A}_{sr} (\mathbf{I} - \mathbf{A}_{rr})^{-1} \mathbf{Y}_{rr} + \mathbf{A}_{sr} (\mathbf{I} - \mathbf{A}_{rr})^{-1} \sum_{j \neq r}^{G} \mathbf{A}_{rj} \sum_{k \neq s}^{G} \mathbf{B}^{\not s}_{jk} \mathbf{Y}_{kr} \right] . \tag{14}$$

$\mathbf{VAXIM}_{sr}$ includes the value-added produced in s and absorbed in r and that factors in the bilateral exports between the two countries. The measure can be used, for instance, in the analysis of bilateral trade balances to assess how reciprocal final demand helps activate bilateral exports and production in the two countries.[8] The $\mathbf{DAVAX}_{sr}$ measure developed within the source-based approach is a subcomponent of the $\mathbf{VAXIM}_{sr}$, as it includes only the value-added produced in s and directly absorbed in r, without any possible re-export (and re-import) by r. Neither the $\mathbf{VAXIM}_{sr}$ nor the $\mathbf{DAVAX}_{sr}$

[8] The $\mathbf{VAXIM}_{sr}$ indicator, as any other measure based on an exporting country-level perspective can be summed across bilateral importing partners to obtain an aggregate indicator for the exporting country s. Then, the $\mathbf{VAXIM}_{sr}$ can be particularly useful, for instance, when we are interested in decomposing the overall trade balance of a country by its bilateral positions. When the analysis focuses exclusively on a given bilateral relation, it might be more appropriate to resort to the approach presented in Section 4.1.

correspond to what Koopman et al. (2014) define as the 'exports absorbed by direct importers.'[9]

Despite having first introduced the concepts of source-based and sink-based approaches, Nagengast and Stehrer (2016) did not propose fully-fledged breakdowns based on these two approaches. On a more general level, Nagengast and Stehrer (2016) focus on the value-added components that are produced and absorbed by the two bilateral partners to evaluate how these items contribute to bilateral trade balances since this is the aim of their analysis. The remaining part of their decomposition does not employ a specific strategy to single out value-added and double-counted terms. Instead, they distinguish between components that belong to the 'domestic content of exports' and those that are part of the 'foreign content of exports' by using expressions that resemble those in Equation (1).[10] Nagengast and Stehrer (2016) also label these terms 'value-added,' instead of 'content,' which can lead to the misinterpretation of results. For instance, their definitions of the domestic value-added finally absorbed at home and in third countries produce an overestimation of the domestic value-added in exports since double-counted components are also included.[11]

Neither methodology proposed by Nagengast and Stehrer (2016) provides a complete break-up of the entire domestic and foreign value-added exported by a country across different bilateral flows. However, this probably goes beyond the scope of their analysis, which focuses on the value-added components produced and absorbed by the two bilateral partners. Since an important aspect of their study is the role of final demand in generating bilateral trade balances, a sink-based approach appears ideally suited. As mentioned, the proper measure of this component is given by the **VAXIM** of equation (14), which differs from the measure proposed by Nagengast and Stehrer (2016).

In general, the following empirical questions can be addressed using a sink-based exporter-perspective decomposition of trade flows:

- What is the value-added breakdown of bilateral trade balances?
- What is the domestic value-added breakdown in bilateral flows traced the last time it crosses the national border, consistent with total export flows?
- What is the foreign value-added breakdown in bilateral flows traced the last time it crosses the national border, consistent with total export flows?

Source-based and sink-based measures can be used in different contexts to address various issues. Nevertheless, it is important to reaffirm that i) at the bilateral level, the domestic and foreign *contents* are the same in the two breakdowns;[12] ii) the 'value-added'

[9] We have already discussed the differences with the $\mathbf{DAVAX}_{sr}$ indicator. The mismeasurement in Koopman et al. (2014) as compared to the $\mathbf{VAXIM}_{sr}$ can be precisely gauged by considering the decomposition of bilateral exports in equation (D.2) in Appendix D. Koopman et al. (2014) allocate the second term of their decomposition to the bilateral importers' final demand; in reality, only sub-items **2a** and **2b** of equation (D.2) can be defined as such (while sub-item **2c** is not). Conversely, part of the third term (**3c**) should also be classified as 'direct importers' final absorption', instead of third countries'.

[10] These components are computed in exactly the same way in the source-based breakdown and sink-based one.

[11] For example in Nagengast and Stehrer (2016) the domestic value-added absorbed in third countries is calculated as $\mathbf{V}_s\mathbf{B}_{ss}\mathbf{A}_{sr}\mathbf{B}_{rr}\sum_{j\neq s,r}\mathbf{Y}_{rj}$. The simultaneous application of the $\mathbf{B}_{ss}$ Leontief inverse matrix and the $\mathbf{B}_{rr}$ one, leads to a double counting of the same value-added.

[12] In both the decompositions the domestic content of exports (i.e. the sum of the domestic value-added and the domestic double counted) corresponds to those defined by Koopman et al. (2010).

and 'double counted' terms of the two decompositions differ only at the bilateral level, and when summing across the destinations of a given exporter, we obtain exactly the same results (see Appendix C for formal proof).

3.3 Exporting country perspective sectoral breakdown

The bilateral decompositions presented above can be easily extended to consider the sectoral dimension. It is worth recalling that here we do not change the perimeter for defining double counting, which is represented by the exporter's borders as a whole. Thus, a unit of value-added first exported by a certain sector and then re-exported by a different sector is accounted for as 'value-added' on one occasion and as 'double counted' on another.[13] By keeping this exporting country-level perspective, we preserve the additivity of value-added components (i.e. the sum across all sectors and all bilateral partners yields the total GDP embedded in a country's exports).[14]

Borin and Mancini (2017) proposed three different sectoral breakdowns: *i*) by origin of the value-added, either domestic or foreign, *ii*) by exports, and *iii*) by final absorption.

To obtain a decomposition by sectors of origin, it is necessary to substitute in all the indicators of Sections 3.2.1 and 3.2.2 the $1 \times N$ vector $\mathbf{V}_j$ $(j = s, t)$ with its diagonalized form $\widehat{\mathbf{V}}_j$ (i.e., the $N \times N$ diagonal matrix with the direct value-added coefficients along the principal diagonal and zeros elsewhere).

Similarly, the decomposition by exporting sectors is obtained by substituting vectors $\mathbf{V}_j\mathbf{B}_{js}$ and $\mathbf{V}_j\mathbf{B}^{\not s}_{js}$ in the DVA and FVA terms of equations (7) and (11) (and the ones that follow) with their $N \times N$ diagonalized forms, $\widehat{\mathbf{V}_j\mathbf{B}_{js}}$ and $\widehat{\mathbf{V}_j\mathbf{B}^{\not s}_{js}}$.

The only exception to this rule is the sectoral breakdown of domestic and foreign double counting in the source-based decomposition. In these cases, the correct diagonalized forms to be substituted are $(\mathbf{V}_s\mathbf{B}^{\not s}_{ss}\widehat{\sum_{j \neq s}\mathbf{A}_{sj}}\mathbf{B}_{js})$ and $(\mathbf{V}_t\mathbf{B}^{\not s}_{ts}\widehat{\sum_{j \neq s}\mathbf{A}_{sj}}\mathbf{B}_{js})$, for domestic and foreign double counting, respectively.

Finally, the decomposition by sectors of final absorption is obtained by replacing the vector of final demand $(\mathbf{Y}_{kl})$ with its $N \times N$ diagonalized form.

Depending on the specific empirical application, it is possible to choose the best suited bilateral sectoral decomposition. For instance, if the focus is on the origin of production, the natural choice is to extend the source-based decomposition with a breakdown by sector of origin. Different sectoral breakdowns can also be combined. For instance, to simultaneously measure the value-added embedded in a bilateral trade flow in a specific sector of origin destined for a particular sector of absorption, we can use $\widehat{\mathbf{V}}_j$ and $\widehat{\mathbf{Y}}_{kl}$ at the same time.

For instance, some empirical questions that can be addressed with the sectoral

[13] Depending on the choice between the source and the sink based decomposition, the 'value-added' is attributed to a certain export flow or to another.

[14] In Section 4.2 we present a decomposition based on a pure sectoral-bilateral perspective which means that a certain unit of value-added is considered as double counted only when it is exported multiple times to the same partner within the same sector. In this framework, the additivity property does not hold, but these alternative measures may be very useful when addressing some specific issues such as trade policy analysis.

breakdown of the exporting country perspective are the following:

- What is the domestic value-added embedded in bilateral-sectoral flows, consistent with total sectoral exports?
- What is the foreign value-added embedded in bilateral sectoral flows, consistent with total sectoral exports?

4 Value-added exposure to shocks: perspectives in line with the selected trade flow

The exporting country-level perspective considered thus far ensures that each component of domestic or foreign value-added corresponds to mutually exclusive portions of the GDP produced by a country and embedded in its own exports, or in another country's exports. In other words, this perspective is additive. This approach can be useful when addressing several empirical questions (see Table 1), and is at the bottom of some key indicators of GVC participation (Borin and Mancini, 2015). However, other potential issues require an accounting framework based on a different perspective.

For instance, this occurs when we want to measure the GDP of a country that passes through a certain trade flow at any point. Suppose, for example, that there is a deterioration in the trade relationship between country s and country r; in this case, we might be interested in measuring the total value-added that crosses this specific bilateral border, regardless of whether the same components are also part of the exports of s (or r) to other countries or not (i.e., they are double counted items from an exporting country-level perspective).

To address this problem, we require an accounting method for value-added in exports that excludes only the value-added components items that are double counted in the same flow from gross trade figures. In this specific case, the bilateral relationship represents the new perimeter for defining double-counted flows in gross exports. The same reasoning can be applied to any level of trade flow aggregation. However, these measures cannot be summed up to obtain value-added in the total exports of a country; that is, they are non-additive. As previously mentioned, by summing these indicators, we obtain value-added measures that exceed the correct measures for aggregate trade flow.

In the following sections, we present several perspectives that are in line with the selected trade flow. These are the bilateral perspective (Section 4.1), sectoral-bilateral perspective (4.2), sectoral-exporter perspective (4.3), importer perspective (4.4), and sectoral-importer perspective (4.5).

4.1 Bilateral-level perspective

By proceeding as for the derivation of the decomposition of aggregate exports (see Section 3.1), we can modify the input coefficient matrix $\mathbf{A}$ to split the production process along the new perimeter and single out the 'value-added' and 'double counted' items. In the exporting country-level perspective, we set the coefficients that identify the direct requirement of intermediate inputs from country s to all the other countries; here, we only set the bilateral coefficient matrix $\mathbf{A}_{sr}$ to zero so that $\mathbf{A}_{jk}^{\cancel{sr}} = \mathbf{A}_{jk}\ \forall jk \neq sr$, while $\mathbf{A}_{sr}^{\cancel{sr}} = 0$. The corresponding inverse Leontief matrix can be defined as: $\mathbf{B}^{\cancel{sr}} = (\mathbf{I} - \mathbf{A}^{\cancel{sr}})^{-1}$.

The submatrix $\mathbf{B}^{\cancel{sr}}_{is}$ measures the gross output produced by i to deliver one unit of production to s, with the exclusion of the production linkages that pass through the bilateral flow s-r; in this way, all the production stages that antecede the export from s to r are taken into account. It is also easy to show that the following relationship holds:

$$\mathbf{B}_{is} = \mathbf{B}^{\cancel{sr}}_{is} + \mathbf{B}^{\cancel{sr}}_{is}\mathbf{A}_{sr}\mathbf{B}_{rs}. \tag{15}$$

By analogy with the derivation of the source-based decomposition in (7), we can express the complete decomposition of bilateral exports based on a purely bilateral perspective:

$$\mathbf{u}_N\mathbf{E}_{sr} = \overbrace{\underbrace{\mathbf{V}_s\mathbf{B}^{\cancel{sr}}_{ss}\mathbf{E}_{sr}}_{\substack{\text{bilateral}\\ \text{perspective}\\ \mathbf{DVA}^*_{sr}}} + \underbrace{\mathbf{V}_s\mathbf{B}^{\cancel{sr}}_{ss}\mathbf{A}_{sr}\mathbf{B}_{rs}\mathbf{E}_{sr}}_{\substack{\text{bilateral}\\ \text{perspective}\\ \mathbf{DDC}^*_{sr}}}}^{\substack{\text{domestic}\\ \text{content } (\mathbf{DC}_{sr})}} + \overbrace{\underbrace{\sum_{t\neq s}^{G}\mathbf{V}_t\mathbf{B}^{\cancel{sr}}_{ts}\mathbf{E}_{sr}}_{\substack{\text{bilateral}\\ \text{perspective}\\ \mathbf{FVA}^*_{sr}}} + \underbrace{\sum_{t\neq s}^{G}\mathbf{V}_t\mathbf{B}^{\cancel{sr}}_{ts}\mathbf{A}_{sr}\mathbf{B}_{rs}\mathbf{E}_{sr}}_{\substack{\text{bilateral}\\ \text{perspective}\\ \mathbf{FDC}^*_{sr}}}}^{\substack{\text{foreign}\\ \text{content } (\mathbf{FC}_{sr})}}. \tag{16}$$

The measures of 'domestic value-added' and 'foreign value-added' in Equation (16) correspond to those proposed by Johnson (2018) in a two-country context; the same measure of 'domestic value-added' in bilateral exports is also obtained by Los et al. (2016) by using a hypothetical extraction procedure (see Table 2 in Section 2).

The components in equation (16) are uniquely defined for a certain bilateral flow s-r, because the perimeter for the definition of double counted items is the bilateral relationship itself.[15]

In Appendix F, we show how they are related to the terms in the bilateral decompositions based on the exporting country-level perspective presented in Section 3. The difference between the exporting country-level perspective applied to bilateral trade flows and the bilateral perspective is also discussed by Los and Timmer (2020), despite the authors considering only the domestic value-added and not the foreign value-added component. Thus, this part of our framework can also be considered as a generalization of the bilateral scheme in Los and Timmer (2020) for both domestic and foreign content.

By following the same scheme as in Section 3.2.1, Equation (16) can be further developed to consider all the forward production linkages, as well as the countries of completion and the markets of final absorption. We can also extend the breakdown to consider the sectors of original production of value-added and ultimate absorption (see Section 3.3).[16]

Some empirical questions that can be addressed with a bilateral perspective are the following:

[15] As for any other case in which the perspective coincides with the considered trade flow, there is no distinction between a source- and sink-based approach. Although Equation (16) was obtained proceeding as for the derivation of the source-based decomposition of Section 3.2.1, we could have obtained the same result by exploiting a sink-based algebra, as in Section 3.2.2.

[16] The decomposition by sector of export, instead, is not univocal even in the bilateral-level perspective and will change depending on whether we employ a source-based or a sink-based approach. However, in this context, we do not consider this type of breakdown particularly meaningful from an economic standpoint. It is more useful to analyze the case in which a specific exporting sector, within a bilateral relationship, is the focus of the analysis.

- What is the GDP potentially exposed to generic trade frictions with another country?
- What is the GDP of other countries, that is, foreign value-added, potentially exposed to generic trade frictions with another country?

4.2 Sectoral-bilateral perspective

It is particularly relevant for some empirical analyses to measure the value-added of a country that enters the exports between two countries in a specific sector. For instance, trade policies are often unilaterally applied by an importing country vis-à-vis a particular export partner and sector (or product) of exports. To assess how this type of policy affects the GDP produced in a given country, we require an accounting framework for the value-added, in which the single sectoral-bilateral flow is the new perimeter used to define the value-added accounting of exports.

By proceeding in the same manner as the other decompositions, we can define a modified version of the input requirement matrix in which all the coefficients corresponding to the intermediate exports from s to r in (exporting) sector n are set to zero ($\mathbf{A}^{\cancel{sr,n}}$). As in previous cases, by employing the corresponding inverse Leontief matrix $\mathbf{B}^{\cancel{sr,n}}$, we obtain a breakdown of exports from a sectoral-bilateral perspective:

$$e_{sr,n} = \overbrace{\underbrace{\mathbf{V}_s\mathbf{B}_{ss}^{\cancel{sr,n}}E_{sr,n}}_{\substack{\text{sectoral-bilateral}\\ \text{perspective}\\ \mathbf{DVA}^{\S}_{sr}}} + \underbrace{\mathbf{V}_s\mathbf{B}_{ss}^{\cancel{sr,n}}\mathbf{A}_{sr}\mathbf{B}_{rs}E_{sr,n}}_{\substack{\text{sectoral-bilateral}\\ \text{perspective}\\ \mathbf{DDC}^{\S}_{sr}}}}^{\substack{\text{domestic}\\ \text{content } (\mathbf{DC}_{sr})}} + \overbrace{\underbrace{\sum_{t\neq s}^{G}\mathbf{V}_t\mathbf{B}_{ts}^{\cancel{sr,n}}E_{sr,n}}_{\substack{\text{bilateral}\\ \text{perspective}\\ \mathbf{FVA}^{\S}_{sr}}} + \underbrace{\sum_{t\neq s}^{G}\mathbf{V}_t\mathbf{B}_{ts}^{\cancel{sr,n}}\mathbf{A}_{sr}\mathbf{B}_{rs}E_{sr,n}}_{\substack{\text{bilateral}\\ \text{perspective}\\ \mathbf{FDC}^{\S}_{sr}}}}^{\substack{\text{foreign}\\ \text{content } (\mathbf{FC}_{sr})}}, \tag{17}$$

where $E_{sr,n}$ is an $N\times 1$ vector with the scalar corresponding to the gross exports from s to r in position n (i.e. $e_{sr,n}$) and zeros elsewhere. As in previous cases, we can consider the forward production-demand linkages up to the market of the final destination as well as a breakdown by sector of origin/final destination, as described in Section 3.3.

Here, the definition of double-counted components is even less restrictive than in the bilateral perspective case, which means the $\mathbf{DVA}^*_{sr}$ and the $\mathbf{FVA}^*_{sr}$ are smaller than the sum of the value-added terms in equation (17) across the different exporting sectors. Notably, this sum does not have an economically meaningful interpretation because of its double-counted components.

Since tariffs are usually imposed at the sectoral-bilateral level, this perspective is suited for evaluating the incidence of tariffs on the value-added produced by different countries. The ICIO framework allows us to single out the countries/sectors that produced the value embedded in the goods on which a tariff is levied. By boosting flows of intermediate goods across countries and stages of production, GVCs tend to amplify the costs generated by border barriers (Rouzet and Miroudot, 2013; Vandenbussche et al. 2017). Since tariffs are applied to gross trade flows, the value-added that crosses the targeted importer border several times is subject to an 'inflated' tariff compared to the original nominal tariff.

Given the nominal tariff $\tau_{sr,n}$ imposed by country r on goods imported from s in sector n, the 'effective tariff' on the value-added produced by country j can be computed as follows:

$$\mathbf{ET}^{j}_{sr,n} = \frac{\mathbf{V}_j \mathbf{B}_{js} E_{sr,n} \tau_{sr,n}}{\mathbf{V}_j \mathbf{B}^{\cancel{sr,n}}_{js} E_{sr,n}} \tag{18}$$

while the incidence of the tariff on the total value-added produced by j can be computed by multiplying $\mathbf{ET}^{j}_{sr,n}$ by the ratio between $\mathbf{V}_j \mathbf{B}^{\cancel{sr,n}}_{js} E_{sr,n}$ and $\mathbf{V}_j \sum_k \sum_l \mathbf{B}_{jk} Y_{kl}$.

Some empirical questions that can be addressed with this perspective are the following:

- What is the GDP potentially exposed to a specific tariff imposed by a trade partner in a given sector?
- What is the GDP of other countries, i.e., foreign value-added in the country's exports, potentially exposed to a specific tariff imposed by a trade partner in a given sector?

4.3 Sectoral-exporter perspective

In some cases, economic shocks or policy measures affect the exports of a country in a given sector, irrespective of the specific bilateral importer. An example is the so-called Dieselgate, involving the main German car producers in 2015. All German exports in the automotive sector became vulnerable to the shock. To assess spillovers from this shock into different countries/sectors, we compute the value-added content embedded in German exports in that specific sector. This can be singled out by deriving a decomposition of a country's aggregate sectoral exports on a 'sectoral-level perspective.'[17]

The domestic and foreign value-added embedded in the total exports of country s and sector n ($\sum_{r \neq s} e_{sr,n}$) can be computed in a manner similar to the 'sectoral-bilateral' decomposition (see equation (17)). The only difference is that the matrix of technical coefficients $\mathbf{A}$ must be modified such that $a_{sj,n}$ is set to zero $\forall\ j \neq s$; thus, the inverse Leontief matrix is computed accordingly.

Some empirical questions that can be addressed with this perspective are the following:

- What is the GDP potentially exposed to a negative foreign demand shock hitting a specific sector?
- What is the GDP of other countries, i.e., foreign value-added in the country's exports, potentially exposed to a negative foreign demand shock hitting a specific sector?

4.4 Importer perspective

Another issue worth considering is when we define the relevant perimeter at the level of the importing country's borders. This 'importer perspective' should be adopted to compute

[17] In this perspective, a certain unit of value-added is considered 'double counted' only when it is re-exported by the same country and sector.

the GDP of a given country j that enters, directly or indirectly, the total imports of a given country r. This measure can be interesting, for instance, when a certain country is going to adopt a general protectionist stance (i.e., vis-à-vis all the exporting partners), and we want to compute the portion of other countries' GDP at stake.

By following a procedure similar to that used to derive the 'exporter perspective' in Section 3, the total gross imports of country r can be decomposed as follows:

$$\mathbf{u}_N \mathbf{E}_{*r} = \underbrace{\sum_{j}^{G} \mathbf{V}_j \sum_{s \neq r} \widetilde{\mathbf{B}}^{\not r}_{js} \mathbf{E}_{sr}}_{\substack{\text{importer}\\ \text{perspective}\\ \text{Value-Added}}} + \underbrace{\sum_{j}^{G} \mathbf{V}_j \sum_{t \neq r} \widetilde{\mathbf{B}}^{\not r}_{jt} \sum_{s \neq r} \mathbf{A}_{tr} \mathbf{B}_{rs} \mathbf{E}_{sr}}_{\substack{\text{importer}\\ \text{perspective}\\ \text{Double-Counted}}} \tag{19}$$

where $\widetilde{\mathbf{B}}^{\not r}_{js}$ is the Leontief inverse block matrix derived from a technical coefficient matrix $\mathbf{A}$ in which the sub-blocks $\mathbf{A}_{tr}$ are set to zero $\forall\ t \neq r$.

In equation (19) we do not use the distinction between 'domestic' and 'foreign' value-added as this notion is related to the exporting country (i.e., the domestic one). However, we can still identify 'value-added' and 'double counted' according to a specific country of origin, labeled j in equation (19).

Some empirical questions that can be addressed with this perspective are the following:

- What is the GDP potentially exposed to trade restrictions vis-à-vis all partners?
- What is the GDP of other countries, i.e., foreign value-added in the country's exports, potentially exposed to trade restrictions vis-à-vis all partners?

4.5 Sectoral-importer perspective

The focus can also be on a particular sector of a given importing country. This occurs when a certain shock affects only the imports of a country of a specific sector, as for the tariff hikes imposed in 2018 by the U.S. administration on steel products imported from all trading partners.

We can easily modify the decomposition in (19) to isolate the value-added generated in a given country j that is embedded in the total imports of a country r in a given sector n. This can be obtained by substituting the matrix $\widetilde{\mathbf{B}}^{\not r}_{js}$ with a Leontief inverse $\widetilde{\mathbf{B}}^{\not r \not n}_{js}$ based on a technical coefficient matrix in which only the elements $a_{tr,n}$ are set to zero $\forall\ t \neq r$.

Clearly, when the focus is on a country's imports from a specific exporting partner, the appropriate decomposition is the one based on the bilateral-level perspective (see Section 4.1) or the sectoral-bilateral one (see Section 4.2) when the bilateral trade flows in the spotlight are only those of a particular sector.

The sectoral-importer accounting perspective can be used to address different empirical questions. Some examples are as follows:

- What is the GDP potentially exposed to a specific sectoral tariff vis-à-vis all partners?

- What is the GDP of other countries, i.e. foreign value-added in the country's exports, potentially exposed to a specific sectoral tariff vis-à-vis all partners?

5 Empirical Examples

In this section, we explore some relevant economic questions that can be addressed precisely using only the accounting framework presented in this work. The first example shows how the indicators derived using the exporting-country perspective source-based approach (see Section 3.2.1) can be used to characterize Chinese exports. Since China is the hub of Factory Asia, a sizable share of its exports consists of foreign production (i.e., processing trade). In addition, within Factory Asia, Chinese inputs are often used and re-exported by partner countries. The rest of Chinese exports, i.e. directly absorbed value-added in exports (DAVAX), is instead a more traditional form of trade, as it crosses just one border. Thanks to the source-based exporting country perspective we are able to provide a quantification of these three terms for Chinese aggregate exports (Section 3.1) and bilateral exports (Section 3.2.1). This might be relevant to better characterize the role of China within Factory Asia and to understand whether trade within the region differs from trade with more distant partners.

In Table 3, we consider the four most relevant Chinese partners according to the WIOD 2014 data and Taiwan, which is a key country within Factory Asia. We find that around two-thirds of the aggregate Chinese exports cross just one border (DAVAX). However, there is substantial heterogeneity at the country level, as the share is much lower for relevant partners within Factory Asia such as Korea (48.5%) and Taiwan (38.1%). In addition, a relevant amount of Chinese intermediates pass through Taiwan to reach other markets, as 42% of Chinese exports to Taiwan consist of Chinese value-added re-exported to other economies. In turn, trade with the US and Germany, which are the hubs of Factory America and Factory Europe, is more traditional, as ties with these countries are less GVC-intensive. The figures for Japan are in line with Chinese aggregate exports, suggesting that the country is for China a final destination market within Asia instead of a production platform, such as Korea and Taiwan.

The source-based exporting country perspective may also be applied to sectoral-bilateral trade flows (see Section 3.3). Indeed, by computing the DAVAX indicator for Chinese exports of computer, electronic, and optical products, we find that it is much lower than the aggregate figures, especially towards Korea (36%) and Taiwan (20%), suggesting that the sector is more GVC-intensive than the average.

In the second example, we assess the extent to which the global economy might be exposed to the deterioration of US-China relations through the lens of ICIO data and our accounting perspectives. A decoupling between the two economies in the form of a high trade barrier on bilateral flows might negatively affect, not only production in both countries, but also economies whose products are embedded in those trade flows. To highlight this aspect, we need to apply a bilateral perspective (Section 4.1), given that the trade shock is between the US and China, and recover not only the domestic value-added, but also the foreign value-added embedded in these export flows, a key feature that is not shared by other works in the literature.[18] In Table 4, we report such an

[18]See also Chen et al. (2018) for an application on Brexit of a regional extension of the Los et al. (2016) bilateral decomposition. Since this decomposition does not cover foreign value-added it would not be suited to assess other countries exposure as we do in this section. Another work on Brexit applying

analysis considering OECD TiVA 2018 data. The Chinese (US) value-added potentially exposed to US (Chinese) trade barriers is around $405 billion ($210 billion). However, other countries' exposure is sizeable, especially to US trade barriers. Indeed, both the US and China are among the most impaired countries by the very same trade barrier they impose, as $8 billion of US value-added is found in Chinese exports to the US, while $3 billion of Chinese value-added is found in US exports to China. In other words, from a trade policy perspective, the analysis highlights that a country that imposes a trade barrier unilaterally might be negatively hit even without a countermeasure from the other country.

In a different scenario, we might assume that a US trade barrier is levied on imports of Chinese computer, electronic, and optical products, that is, a sort of tech decoupling. To analyze the potential effect of such a shock, we need to resort to the sectoral-bilateral perspective (Section 4.2). The Chinese value-added potentially exposed to this US unilateral tariff is $99.7 billion. This is higher by around $2 billion than the Chinese value-added traced in the same export flow using an exporting-country perspective. Thus, if we had used the latter perspective, we would have understated the Chinese exposure. This finding supports the view that each specific economic question calls for the better-suited perspective within our framework.

In the third example, we consider the potential impact of a sizable and pervasive drop in Russian imports following the start of the war in Ukraine. Trade data from March and April 2022 suggest that export bans from sanctioning countries, collapse in Russian domestic demand, private firms' boycotts, financial sanctions, and logistic disruptions have remarkably impaired Russian imports vis-à-vis all partners. Thanks to the importer perspective (Section 4.4) and the recent ADB MRIO data for 2019, we can determine which countries are more exposed to this specific shock. As reported in Table 5, a sharp drop in Russian imports may have a significant impact on the economies of Central Asian countries. The exposure of Baltic countries is sizable, as around 4% of their GDP is activated by Russian imports. This corresponds to around 8% of their total GDP absorbed abroad. Even if the exposure to the shock of G20 economies is more modest, losses might be non-negligible for major Euro area economies, such as Germany (1.1% of GDP) and Italy (0.7%).

Table 3: Chinese exports to selected partners

	Gross exports ($mln)	DAVAX (share of exports)	Re-exported DVA (share of exports)	Import content of exports (share of exports)
United States	347'302	75.9%	5.4%	18.7%
Japan	172'857	69.9%	11.8%	18.2%
Korea	101'921	48.5%	32.4%	19.2%
Germany	88'462	60.7%	21.6%	17.7%
Taiwan	43'621	38.1%	41.1%	20.8%
Total	*2'425'406*	*67.8%*	*15.4%*	*16.9%*

the aforementioned decomposition is IJtsma et al. (2018).

Table 4: GDP exposure to China-US trade war

	US trade barrier on imports from CHN ($mln)	(% total exposed VA)	CHN trade barrier on imports from US	($mln)	(% total exposed VA)
China	405'359	82.7%	United States	210'509	95.5%
Korea	11'346	2.3%	China	3'170	1.4%
United States	8'490	1.7%	Canada	2'546	1.2%
Japan	8'164	1.7%	Mexico	1'842	0.8%
Taiwan	6'154	1.3%	Japan	1'203	0.5%
Germany	3'974	0.8%	Germany	1'122	0.5%
Others	46'839	9.5%	Others	10'935	5.0%

Table 5: GDP exposure to disruptions in Russian imports

	GDP exposure (% of total GDP)	(% of GDP absorbed abroad)	($mln)
Overall most exposed			
Kazakhstan	9.3%	28.0%	14'964
Mongolia	4.2%	8.7%	553
Lithuania	4.1%	8.5%	2'072
Estonia	3.9%	8.3%	1'134
Latvia	3.5%	8.1%	1'104
Kyrgyzstan	3.2%	11.8%	222
Among G20 countries			
Turkey	1.2%	5.2%	8'651
Germany	1.1%	3.3%	38'206
Italy	0.7%	2.8%	12'596
UK	0.7%	2.5%	17'516
Korea	0.7%	2.4%	10'356
France	0.5%	2.2%	13'133

6 Concluding remarks

The diffusion of international production networks over the last three decades calls for new tools to evaluate supply and demand relationships between countries which can no longer be adequately gauged by gross trade flows. Therefore, ICIO tables and new methodologies that exploit these data have been developed to measure trade in value-added terms and countries' participation in GVCs. These tools have been used extensively in numerous applications. However, on the one hand, several empirical issues have not been properly addressed thus far; on the other hand, the emergence of different methodologies for value-added accounting of trade flows has raised doubts regarding the correct way of measuring the phenomena.

In this paper, we proposed a general scheme for breaking down aggregate, bilateral, and sectoral export flows according to the source and destination of their value-added content. This framework can be differentiated according to alternative approaches and perspectives, which are instrumental in addressing different empirical issues. These different approaches are reflected in distinct ways in distinguishing between 'value-added' and 'double-counted' terms in gross trade flows. Operationally, this differentiation was obtained by changing the definition of sectoral-geographical perimeters according to which the value-added units that cross these boundaries more than once are classified as 'double counted.' We considered three alternative cases in more detail: in the first case, the boundaries were defined at the level of the exporting country; in the second case, at the level of a specific bilateral trade relationship; and in the third case, at the level of a single exporting sector within a bilateral flow. We also derived accounting methods for other relevant perspectives. We argued that each perspective is conceptually suited to address a different class of empirical issues. For instance, the exporting country-level approach is appropriate when allocating a country's GDP across different trade flows, whereas the bilateral perspective is best suited to evaluating the extent to which a country's GDP is involved in the commercial interchange between the two economies. The framework proposed in this paper is implemented and ready-to-use in the Stata command *icio* (see Belotti, Borin and Mancini, 2021).

We showed how the main methodologies proposed in other studies fit this framework. On the one hand, we reconciled a large part of the existing literature under a unique comprehensive scheme; on the other hand, we highlighted the main shortcomings and limitations that affect some of the techniques proposed in the literature. Moreover, we extended the set of possible measures to address a wider range of empirical issues. These improvements are likely to become increasingly relevant from a quantitative point of view because the ICIO data will become increasingly detailed, eventually relaxing some of the simplifying assumptions that characterize current databases (de Gortari, 2018).

Finally, an aspect that is currently debated in the literature is how to measure the impact of trade policies on countries' productions by taking all international supply linkages into account. The novel indicators proposed in this paper may contribute to measuring the spillovers of trade policies in global value chains using ICIO data. For instance, by exploiting the characteristics of the 'sectoral-bilateral' perspective, we derived an indicator of the effective incidence of tariffs on value added. Through numerical examples, we also showed how the proposed measures can be used to assess the effects of a trade war between China and the United States, or the decoupling of Russia from other economies.

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A Appendix: Notation and basic I-O relations

This appendix simply recalls our notation, which is broadly the same as Koopman et al. (2014), together with some basic accounting relationships.

We consider the general case of G countries producing N goods that are internationally traded both as intermediate inputs and as final goods. Thus, $\mathbf{X}_s = (x_1^s \; x_2^s \; \cdots \; x_N^s)'$ is the $N \times 1$ vector of the gross output of country s and Y_s is the $N \times 1$ vector of final goods, which is equal to the final demand for goods produced in s in each country of destination r: $\sum_r^G \mathbf{Y}_{sr}$. To produce one unit of gross output of good i a country uses a certain amount a of intermediate good j produced at home or imported from other countries. Thus, each unit of gross output can be either consumed as a final good or used as an intermediate good at home or abroad:

$$\mathbf{X}_s = \sum_r^G (\mathbf{A}_{sr}\mathbf{X}_r + \mathbf{Y}_{sr})$$

where $\mathbf{A}_{sr}$ is the $N \times N$ matrix of coefficients for intermediate inputs produced in s and processed further in r:

$$\mathbf{A}_{sr} = \begin{bmatrix} a_{sr,11} & a_{sr,12} & \cdots & a_{sr,1N} \\ a_{sr,21} & a_{sr,22} & \cdots & a_{sr,2N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{sr,N1} & a_{sr,N2} & \cdots & a_{sr,NN}. \end{bmatrix}$$

Using the block matrix notation, the general setting of production and trade with G countries and N goods can be expressed as follows:

$$\underset{(NG\times 1)}{\begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \\ \vdots \\ \mathbf{X}_G \end{bmatrix}} = \underset{(NG\times NG)}{\begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} & \cdots & \mathbf{A}_{1G} \\ \mathbf{A}_{21} & \mathbf{A}_{22} & \cdots & \mathbf{A}_{2G} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{A}_{G1} & \mathbf{A}_{G2} & \cdots & \mathbf{A}_{GG} \end{bmatrix}} \underset{(NG\times 1)}{\begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \\ \vdots \\ \mathbf{X}_G \end{bmatrix}} + \underset{(NG\times G)}{\begin{bmatrix} \mathbf{Y}_{11} & \mathbf{Y}_{12} & \cdots & \mathbf{Y}_{1G} \\ \mathbf{Y}_{21} & \mathbf{Y}_{22} & \cdots & \mathbf{Y}_{2G} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{Y}_{G1} & \mathbf{Y}_{G2} & \cdots & \mathbf{Y}_{GG} \end{bmatrix}} \underset{(G\times 1)}{\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}} \tag{A.1}$$

from which it is straightforward to derive the following relationship between gross output and final demand:

$$\begin{aligned} \begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \\ \vdots \\ \mathbf{X}_G \end{bmatrix} &= \begin{bmatrix} \mathbf{I}-\mathbf{A}_{11} & -\mathbf{A}_{12} & \cdots & -\mathbf{A}_{1G} \\ -\mathbf{A}_{21} & \mathbf{I}-\mathbf{A}_{22} & \cdots & -\mathbf{A}_{2G} \\ \vdots & \vdots & \ddots & \vdots \\ -\mathbf{A}_{G1} & -\mathbf{A}_{G2} & \cdots & \mathbf{I}-\mathbf{A}_{GG} \end{bmatrix}^{-1} \begin{bmatrix} \sum_r^G \mathbf{Y}_{1r} \\ \sum_r^G \mathbf{Y}_{2r} \\ \vdots \\ \sum_r^G \mathbf{Y}_{1G} \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{B}_{11} & \mathbf{B}_{12} & \cdots & \mathbf{B}_{1N} \\ \mathbf{B}_{21} & \mathbf{B}_{22} & \cdots & \mathbf{B}_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{B}_{G1} & \mathbf{B}_{G2} & \cdots & \mathbf{B}_{GG} \end{bmatrix} \begin{bmatrix} \sum_r^G \mathbf{Y}_{1r} \\ \sum_r^G \mathbf{Y}_{2r} \\ \vdots \\ \sum_r^G \mathbf{Y}_{1G} \end{bmatrix} \end{aligned} \tag{A.2}$$

where $\mathbf{B}_{sr}$ denotes the $N \times N$ block of the Leontief inverse matrix in a global IO setting. It indicates how much of country s's gross output of a certain good is required to produce one unit of country r's final production.

The direct value-added share in each unit of gross output produced by country s is equal to one minus the sum of the direct intermediate input share of all the domestic and foreign suppliers:

$$\mathbf{V}_s = \mathbf{u}_N(\mathbf{I} - \sum_r^G \mathbf{A}_{rs}) \tag{A.3}$$

where $\mathbf{u}_N$ is the $1{\times}N$ unit row vector. Thus, the $G{\times}GN$ direct domestic value-added matrix for all countries can be defined as:

$$\mathbf{V} = \begin{bmatrix} \mathbf{V}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{V}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{V}_G \end{bmatrix}$$

while the overall $G{\times}GN$ value-added share matrix is obtained by multiplying the $\mathbf{V}$ matrix by the Leontief inverse $\mathbf{B}$:

$$\mathbf{VB} = \begin{bmatrix} \mathbf{V}_1\mathbf{B}_{11} & \mathbf{V}_1\mathbf{B}_{12} & \cdots & \mathbf{V}_1\mathbf{B}_{1G} \\ \mathbf{V}_2\mathbf{B}_{21} & \mathbf{V}_2\mathbf{B}_{22} & \cdots & \mathbf{V}_2\mathbf{B}_{2G} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{V}_G\mathbf{B}_{G1} & \mathbf{V}_G\mathbf{B}_{G2} & \cdots & \mathbf{V}_G\mathbf{B}_{GG} \end{bmatrix}$$

Since the value-added shares of different countries in final goods have to sum to one, the following property holds:

$$\sum_t^G \mathbf{V}_t\mathbf{B}_{tr} = \mathbf{u}_N \tag{A.4}$$

Defining the $GN{\times}G$ final demand matrix as:

$$\mathbf{Y} = \begin{bmatrix} \mathbf{Y}_{11} & \mathbf{Y}_{12} & \cdots & \mathbf{Y}_{1G} \\ \mathbf{Y}_{21} & \mathbf{Y}_{22} & \cdots & \mathbf{Y}_{2G} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{Y}_{G1} & \mathbf{Y}_{G2} & \cdots & \mathbf{Y}_{GG} \end{bmatrix}$$

we can derive the $G{\times}G$ value-added matrix by pairs of source-absorption countries:

$$\overline{\mathbf{VA}} \equiv \mathbf{VBY} =$$

$$= \begin{bmatrix} \mathbf{V}_1 \sum_r^G \mathbf{B}_{1r}\mathbf{Y}_{r1} & \mathbf{V}_1 \sum_r^G \mathbf{B}_{1r}\mathbf{Y}_{r2} & \cdots & \mathbf{V}_1 \sum_r^G \mathbf{B}_{1r}\mathbf{Y}_{rG} \\ \mathbf{V}_2 \sum_r^G \mathbf{B}_{2r}\mathbf{Y}_{r2} & \mathbf{V}_2 \sum_r^G \mathbf{B}_{2r}\mathbf{Y}_{r2} & \cdots & \mathbf{V}_2 \sum_r^G \mathbf{B}_{2r}\mathbf{Y}_{rG} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{V}_G \sum_r^G \mathbf{B}_{Gr}\mathbf{Y}_{rG} & \mathbf{V}_G \sum_r^G \mathbf{B}_{Gr}\mathbf{Y}_{rG} & \cdots & \mathbf{V}_G \sum_r^G \mathbf{B}_{Gr}\mathbf{Y}_{rG} \end{bmatrix} \tag{A.5}$$

In Section

To get a decomposition by sectors of origin, it is necessary to substitute in all the indicators of Sections 4.2.1 and 4.2.2 the $1{\times}N$ vector $\mathbf{V}_j$ $(j = s, t)$ with its diagonalized form $\widehat{\mathbf{V}}_j$ (i.e. the $N{\times}N$ diagonal matrix with the direct value-added coefficients along the principal diagonal and zeros elsewhere):

$$\widehat{\mathbf{V}}_j \equiv \begin{bmatrix} v_{j,1} & 0 & \cdots & 0 \\ 0 & v_{j,2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & v_{j,N} \end{bmatrix}.$$

Similarly, the decomposition by exporting sectors is obtained by substituting vectors $\mathbf{V}_j\mathbf{B}_{js}$ and $\mathbf{V}_j\mathbf{B}^{\not s}_{js}$ in the DVA and FVA terms of equations (7) and (11) (and the following ones) with their $N{\times}N$ diagonalized forms, $\widehat{\mathbf{V}_j\mathbf{B}_{js}}$ and $\widehat{\mathbf{V}_j\mathbf{B}^{\not s}_{js}}$:[19]

$$\widehat{\mathbf{V}_j\mathbf{B}_{js}} \equiv \begin{bmatrix} \sum_n^N v_{j,n}b_{js,n1} & 0 & \cdots & 0 \\ 0 & \sum_n^N v_{j,n}b_{js,n2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sum_n^N v_{j,n}b_{js,nN} \end{bmatrix}.$$

Finally the decomposition by sectors of final absorption is obtained by replacing the vector of final demand with its diagonalized form. For instance, for goods completed in country k and absorbed in country l, the $N{\times}N$ diagonal matrix of final demand is as follows:

$$\widehat{\mathbf{Y}}_{kl} \equiv \begin{bmatrix} y_{kl,1} & 0 & \cdots & 0 \\ 0 & y_{kl,2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & y_{kl,N\cdot} \end{bmatrix}.$$

[19]The only exception to this rule is the sectoral breakdown of the domestic and foreign double counting in the source-based decomposition. In these cases, the correct diagonalized forms to be substituted are $(\widehat{\mathbf{V}_s\mathbf{B}^{\not s}_{ss}\sum_{j\neq s}\mathbf{A}_{sj}}\mathbf{B}_{js})$ and $(\widehat{\mathbf{V}_t\mathbf{B}^{\not s}_{ts}\sum_{j\neq s}\mathbf{A}_{sj}}\mathbf{B}_{js})$, respectively for the domestic and foreign double counting.

B Appendix: Useful equivalences in ICIO models

The following two equivalences are often used in ICIO modeling to put different objects in relation to each other.

Considering the following property of inverse matrix B:

$$\mathbf{B}(\mathbf{I}-\mathbf{A}) = (\mathbf{I}-\mathbf{A})\mathbf{B} = \mathbf{I}$$

it is easily shown that the generic block diagonal element $\mathbf{B}_{ss}$ may be expressed as follows:

$$\begin{aligned}\mathbf{B}_{ss} &= \sum_{t\neq s}^{G}\mathbf{B}_{st}\mathbf{A}_{ts}(\mathbf{I}-\mathbf{A}_{ss})^{-1} + (\mathbf{I}-\mathbf{A}_{ss})^{-1} = \\ &= (\mathbf{I}-\mathbf{A}_{ss})^{-1} + (\mathbf{I}-\mathbf{A}_{ss})^{-1}\sum_{t\neq s}^{G}\mathbf{A}_{st}\mathbf{B}_{ts} \qquad \text{(B.1)}\end{aligned}$$

while the generic off-diagonal block element $\mathbf{B}_{rs}$ corresponds to:

$$\begin{aligned}\mathbf{B}_{rs} &= \sum_{t\neq s}^{G}\mathbf{B}_{rt}\mathbf{A}_{ts}(\mathbf{I}-\mathbf{A}_{ss})^{-1} = \\ &= (\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{t\neq r}^{G}\mathbf{A}_{rt}\mathbf{B}_{ts} \qquad \text{(B.2)}\end{aligned}$$

In deriving our bilateral decompositions, we introduce a modified version of the Leontief inverse matrix, $\mathbf{B}^{\not s}$. Here we show some key relationships between the elements of this matrix and those of the 'traditional' Leontief inverse matrix ($\mathbf{B}$) and the input requirement matrix ($\mathbf{A}$).

First, we recall that $\mathbf{B}^{\not s}$ is obtained by setting equal to 0 the coefficients that identify the requirement of inputs from country s in the $\mathbf{A}$ matrix (excepting only the domestic input requirement matrix $\mathbf{A}_{ss}$). Thus, the modified matrix of input requirements can be expressed as follows:

$$\mathbf{A}^{\not s} = \mathbf{A} - \mathbf{A}^{s} \qquad \text{(B.3)}$$

where $\mathbf{A}^{s}$ is the $GN\times GN$ matrix with the coefficients of intermediate inputs imported from s in the corresponding sub-matrices and zero elsewhere. Since $\mathbf{B}^{\not s}$ is the inverse of $(\mathbf{I}-\mathbf{A}^{\not s})$, the following relationships hold:

$$(\mathbf{I}-\mathbf{A}^{\not s})\mathbf{B}^{\not s} = \mathbf{B}^{\not s}(\mathbf{I}-\mathbf{A}^{\not s}) = \mathbf{I} \qquad \text{(B.4)}$$

Substituting (B.3) into (B.4) we get:

$$(\mathbf{I}-\mathbf{A})\mathbf{B}^{\not s} + \mathbf{A}^{s}\mathbf{B}^{\not s} = \mathbf{B}^{\not s}(\mathbf{I}-\mathbf{A}) + \mathbf{B}^{\not s}\mathbf{A}^{s} = \mathbf{I} \qquad \text{(B.5)}$$

and multiplying both sides of (B.5) by $\mathbf{B} \equiv (\mathbf{I}-\mathbf{A})^{-1}$ we obtain the following equivalence:

$$\mathbf{B} = \mathbf{B}^{\not s} + \mathbf{B}\mathbf{A}^{s}\mathbf{B}^{\not s} = \mathbf{B}^{\not s} + \mathbf{B}\mathbf{A}^{s}\mathbf{B}^{\not s} \qquad \text{(B.6)}$$

Then we focus on the off-diagonal block element $\mathbf{B}_{sr}$ that identifies the gross output generated in s necessary to produce one unit of r final good. According to equation (B.6) this sub-matrix can be expressed as follows:

$$\mathbf{B}_{sr} = \mathbf{B}^{\not s}_{sr} + \mathbf{B}_{ss}\sum_{t\neq s}\mathbf{A}_{st}\mathbf{B}^{\not s}_{tr} \qquad \text{(B.7)}$$

where $\mathbf{B}^{\not\sharp}_{sr}$ is equal to $\mathbf{0}$ for each $r \neq s$, since it corresponds to a summation of infinite terms all equal to the null matrix. Therefore, if we single out the $\mathbf{B}^{\not\sharp}_{rr}$ element from the final summation of the right-hand side of equation (B.7) we get:

$$\mathbf{B}_{sr} = \mathbf{B}_{ss}\mathbf{A}_{sr}\mathbf{B}^{\not\sharp}_{rr} + \mathbf{B}_{ss}\sum_{t\neq s,r}\mathbf{A}_{st}\mathbf{B}^{\not\sharp}_{tr} \tag{B.8}$$

Then applying to the elements of matrix $\mathbf{B}^{\not\sharp}$ the properties of $\mathbf{B}$ sub-matrices illustrated in (B.1) and (B.2):

$$\mathbf{B}^{\not\sharp}_{rr} = (\mathbf{I} - \mathbf{A}_{rr})^{-1} + (\mathbf{I} - \mathbf{A}_{rr})^{-1}\sum_{j\neq r}\mathbf{A}_{rj}\mathbf{B}^{\not\sharp}_{jr} \tag{B.9}$$

$$\mathbf{B}^{\not\sharp}_{tr} = (\mathbf{I} - \mathbf{A}_{tt})^{-1}\sum_{j\neq t}^{G}\mathbf{A}_{tj}\mathbf{B}^{\not\sharp}_{jr} \tag{B.10}$$

C Appendix: Proofs

Equivalence between the domestic value-added source-based and the complement of the 'import content of exports' in Hummels et al. (2001)

Here we show that the import-content of exports indicator ($\mathbf{VS}_{s*}$) proposed by Hummels et al. (2001) is equal to the complement of the domestic value-added ($\mathbf{DVA}source_{s*}$) in equation (4). Hummels et al. (2001) claim that the import-content in the total exports of country s can be measured as:

$$\mathbf{VS}_{s*} = \mathbf{u}_N \sum_{j \neq s} \mathbf{A}_{js}(\mathbf{I} - \mathbf{A}_{ss})^{-1}\mathbf{E}_{s*} \tag{C.1}$$

which we will prove to be equal to the following expression:

$$\mathbf{V}_s\mathbf{B}_{ss}^{\not s} \sum_{j \neq s} \mathbf{A}_{sj}\mathbf{B}_{js}\mathbf{E}_{s*} + \sum_{t \neq s} \mathbf{V}_t\mathbf{B}_{ts}^{\not s}\mathbf{E}_{s*} + \sum_{t \neq s} \mathbf{V}_t\mathbf{B}_{ts}^{\not s} \sum_{j \neq s} \mathbf{A}_{sj}\mathbf{B}_{js}\mathbf{E}_{s*} \tag{C.2}$$

Since $\mathbf{u}_N = \sum_t \mathbf{V}_t\mathbf{B}_{tj}$ (see (A.4)), we can rewrite the expression in (C.1) as:

$$\mathbf{V}_s\mathbf{B}_{sj} \sum_{j \neq s} \mathbf{A}_{js}(\mathbf{I} - \mathbf{A}_{ss})^{-1}\mathbf{E}_{s*} + \sum_{t \neq s} \mathbf{V}_t\mathbf{B}_{tj} \sum_{j \neq s} \mathbf{A}_{js}(\mathbf{I} - \mathbf{A}_{ss})^{-1}\mathbf{E}_{s*} \tag{C.3}$$

While, from equation (3) it follows that the expression in (C.2) is equal to:

$$\mathbf{V}_s(\mathbf{I} - \mathbf{A}_{ss})^{-1} \sum_{j \neq s} \mathbf{A}_{sj}\mathbf{B}_{js}\mathbf{E}_{s*} + \sum_{t \neq s} \mathbf{V}_t\mathbf{B}_{ts}\mathbf{E}_{s*}, \tag{C.4}$$

where in (C.4) we make use of the equivalence between $\mathbf{B}_{ss}^{\not s}$ and $(\mathbf{I} - \mathbf{A}_{ss})^{-1}$.

From equivalence in (B.1) we know that:

$$\mathbf{V}_s\mathbf{B}_{sj} \sum_{j \neq s} \mathbf{A}_{js}(\mathbf{I} - \mathbf{A}_{ss})^{-1}\mathbf{E}_{s*} = \mathbf{V}_s(\mathbf{I} - \mathbf{A}_{ss})^{-1} \sum_{j \neq s} \mathbf{A}_{sj}\mathbf{B}_{js}\mathbf{E}_{s*} \tag{C.5}$$

while, from (B.2)

$$\sum_{t \neq s} \mathbf{V}_t\mathbf{B}_{tj} \sum_{j \neq s} \mathbf{A}_{js}(\mathbf{I} - \mathbf{A}_{ss})^{-1}\mathbf{E}_{s*} = \sum_{t \neq s} \mathbf{V}_t\mathbf{B}_{ts}\mathbf{E}_{s*} \tag{C.6}$$

Then, it is proved that the expression in (C.1) is equal to that in (C.2). Q.E.D.

Equivalence between the source-based value-added in exports and the sink-based one for aggregate exports of a country

The domestic (foreign) value-added in the aggregate exports of country s can be obtained by summing across the bilateral importers r the expressions in equation (G.1) (equation

(G.3)), for the source-based approach, and in equation (G.7) (equation (G.9)), for the sink-based one. Simplifying the expressions by abstaining from keeping trace of the country of re-export, the total value-added originated in a given country j and exported by country s can be expressed as follows:

$$\mathbf{VA}source^j_s = \mathbf{V}_j\mathbf{B}^{\nexists}_{js}\sum_{r\neq s}^{G}\mathbf{Y}_{sr} + \mathbf{V}_j\mathbf{B}^{\nexists}_{js}\sum_{r\neq s}^{G}\mathbf{A}_{sr}\sum_{k}^{G}\sum_{l}^{G}\mathbf{B}_{jk}\mathbf{Y}_{kl} \tag{C.7}$$

$$\mathbf{VA}sink^j_s = \mathbf{V}_j\mathbf{B}_{js}\sum_{r\neq s}^{G}\mathbf{Y}_{sr} + \mathbf{V}_j\mathbf{B}_{js}\sum_{r\neq s}^{G}\mathbf{A}_{sr}\mathbf{B}^{\nexists}_{rs}\mathbf{Y}_{ss} + \mathbf{V}_j\mathbf{B}_{js}\sum_{r\neq s}^{G}\mathbf{A}_{sr}\sum_{k\neq s}^{G}\sum_{l}^{G}\mathbf{B}^{\nexists}_{rk}\mathbf{Y}_{kl} \tag{C.8}$$

.

From equation (B.6) it follows that $\mathbf{B}_{js} = \mathbf{B}^{\nexists}_{js} + \mathbf{B}^{\nexists}_{js}\sum_{r\neq s}^{G}\mathbf{A}_{sr}\mathbf{B}_{rs}$; then equation (C.8) can be re-expressed as:

$$\begin{aligned}\mathbf{VA}sink^j_s = {} & \mathbf{V}_j\mathbf{B}^{\nexists}_{js}\sum_{r\neq s}^{G}\mathbf{Y}_{sr} + \mathbf{V}_j\mathbf{B}^{\nexists}_{js}\sum_{r\neq s}^{G}\mathbf{A}_{sr}\mathbf{B}_{rs}\sum_{l\neq s}^{G}\mathbf{Y}_{sl} + \mathbf{V}_j\mathbf{B}_{js}\sum_{r\neq s}^{G}\mathbf{A}_{sr}\mathbf{B}^{\nexists}_{rs}\mathbf{Y}_{ss} \\ & + \mathbf{V}_j\mathbf{B}_{js}\sum_{r\neq s}^{G}\mathbf{A}_{sr}\sum_{k\neq s}^{G}\sum_{l}^{G}\mathbf{B}^{\nexists}_{rk}\mathbf{Y}_{kl},\end{aligned} \tag{C.9}$$

where we used the equivalence $\sum_{l\neq s}^{G}\mathbf{Y}_{sl} \equiv \sum_{r\neq s}^{G}\mathbf{Y}_{sr}$.

Since $\mathbf{B}_{js}\sum_{r\neq s}^{G}\mathbf{A}_{sr}\mathbf{B}^{\nexists}_{rk} = \mathbf{B}^{\nexists}_{js}\sum_{r\neq s}^{G}\mathbf{A}_{sr}\mathbf{B}_{rk}$, we can re-write equation (C.9) as:

$$\mathbf{VA}sink^j_s = \mathbf{V}_j\mathbf{B}^{\nexists}_{js}\sum_{r\neq s}^{G}\mathbf{Y}_{sr} + \mathbf{V}_j\mathbf{B}^{\nexists}_{js}\sum_{r\neq s}^{G}\mathbf{A}_{sr}\sum_{k}^{G}\sum_{l}^{G}\mathbf{B}_{jk}\mathbf{Y}_{kl} \tag{C.10}$$

which is equal to the value-added originated in j and exported by s according to the source-based approach of equation (C.7), Q.E.D.

D Appendix: Comparison with Koopman et al. (2014) and other related contributions

Koopman et al. (2014) proposed the first accounting framework to single out the entire domestic and foreign value-added embedded in the aggregate exports of country s, as well as the double counted components originally produced at home and abroad. In particular, they highlight that some trade flows are purely double-counted, such as when intermediate inputs cross a country's borders several times during the different stages of production. Figure 2 shows a simplified scheme of their breakdown of aggregate exports. Notice that $\mathbf{VAX}_s$ proposed by Johnson and Noguera (2012) is a subcomponent of the domestic value-added embedded in gross exports, the remaining part being the value-added that is finally absorbed by the exporting country itself (labeled 'Reflection' by Koopman et al., 2014).

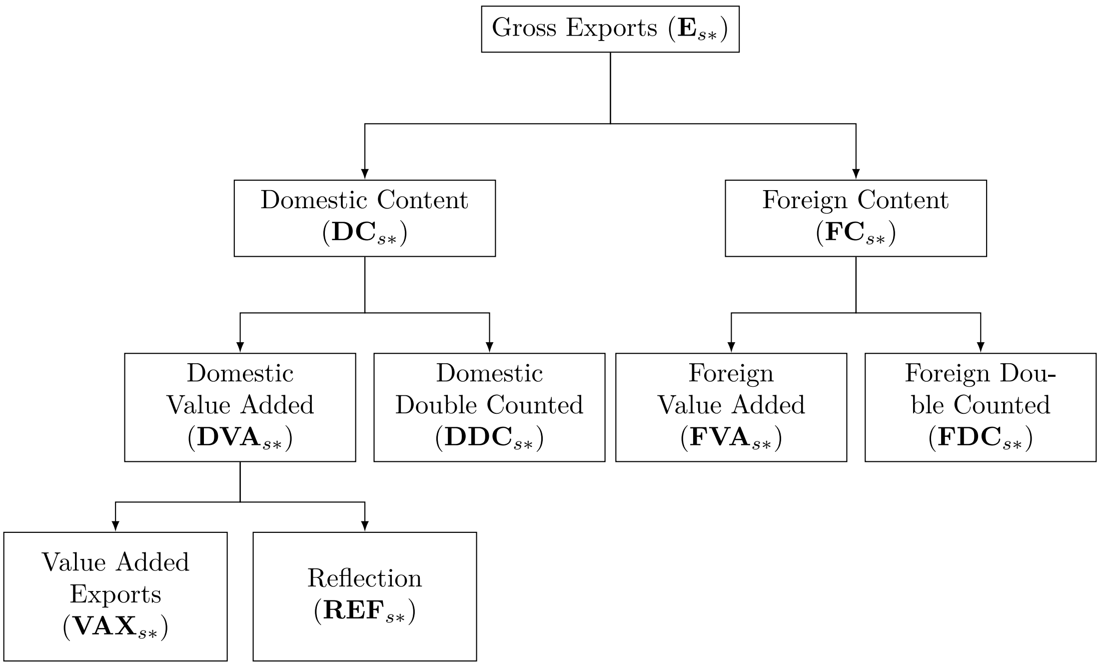

Figure 2: A basic scheme of the Koopman et al. (2014) decomposition of total exports

The Koompan et al. (2014) decomposition of total exports of country s ($\mathbf{u}_N\mathbf{E}_{s*}$) is summarized by the following accounting relationship:

$$
\begin{aligned}
\mathbf{u}_N\mathbf{E}_{s*} &= \left\{ \mathbf{V}_s \sum_{r\neq s}^{G} \mathbf{B}_{ss}\mathbf{Y}_{sr} + \mathbf{V}_s \sum_{r\neq s}^{G} \mathbf{B}_{sr}\mathbf{Y}_{rr} + \mathbf{V}_s \sum_{r\neq s}^{G} \sum_{t\neq s,r}^{G} \mathbf{B}_{sr}\mathbf{Y}_{rt} \right\} \\
&+ \left\{ \mathbf{V}_s \sum_{r\neq s}^{G} \mathbf{B}_{sr}\mathbf{Y}_{rs} + \mathbf{V}_s \sum_{r\neq s}^{G} \mathbf{B}_{sr}\mathbf{A}_{rs}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{Y}_{ss} \right\} \\
&+ \mathbf{V}_s \sum_{r\neq s}^{G} \mathbf{B}_{sr}\mathbf{A}_{rs}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\mathbf{E}_{s*} \\
&+ \left\{ \sum_{t\neq s}^{G} \sum_{r\neq s}^{G} \mathbf{V}_t\mathbf{B}_{ts}\mathbf{Y}_{sr} + \sum_{t\neq s}^{G} \sum_{r\neq s}^{G} \mathbf{V}_t\mathbf{B}_{ts}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{Y}_{rr} \right\}
\end{aligned}
$$

$$+ \quad \sum_{t \neq s}^{G} \sum_{r \neq s}^{G} \mathbf{V}_t \mathbf{B}_{ts} \mathbf{A}_{sr} (\mathbf{I} - \mathbf{A}_{rr})^{-1} \mathbf{E}_{r*} \tag{D.1}$$

KWW defines the nine items in equation (D.1) as follows:

1. $\mathbf{V}_s \sum_{r \neq s}^{G} \mathbf{B}_{ss} \mathbf{Y}_{sr}$: domestic value-added in direct final goods exports;
2. $\mathbf{V}_s \sum_{r \neq s}^{G} \mathbf{B}_{sr} \mathbf{Y}_{rr}$: domestic value-added in intermediate exports absorbed by direct importers;
3. $\mathbf{V}_s \sum_{r \neq s}^{G} \sum_{t \neq s,r}^{G} \mathbf{B}_{sr} \mathbf{Y}_{rt}$: domestic value-added in intermediate goods re-exported to third countries;
4. $\mathbf{V}_s \sum_{r \neq s}^{G} \mathbf{B}_{sr} \mathbf{Y}_{rs}$: domestic value-added in intermediate exports reimported as final goods;
5. $\mathbf{V}_s \sum_{r \neq s}^{G} \mathbf{B}_{sr} \mathbf{A}_{rs} (\mathbf{I} - \mathbf{A}_{ss})^{-1} \mathbf{Y}_{ss}$: domestic value-added in intermediate inputs reimported as intermediate goods and finally absorbed at home;
6. $\mathbf{V}_s \sum_{r \neq s}^{G} \mathbf{B}_{sr} \mathbf{A}_{rs} (\mathbf{I} - \mathbf{A}_{ss})^{-1} \mathbf{E}_{s*}$: double-counted intermediate exports originally produced at home;
7. $\sum_{t \neq s}^{G} \sum_{r \neq s}^{G} \mathbf{V}_t \mathbf{B}_{ts} \mathbf{Y}_{sr}$: foreign value-added in exports of final goods;
8. $\sum_{t \neq s}^{G} \sum_{r \neq s}^{G} \mathbf{V}_t \mathbf{B}_{ts} \mathbf{A}_{sr} (\mathbf{I} - \mathbf{A}_{rr})^{-1} \mathbf{Y}_{rr}$: foreign value-added in exports of intermediate goods;
9. $\sum_{t \neq s}^{G} \sum_{r \neq s}^{G} \mathbf{V}_t \mathbf{B}_{ts} \mathbf{A}_{sr} (\mathbf{I} - \mathbf{A}_{rr})^{-1} \mathbf{E}_{r*}$: double-counted intermediate exports originally produced abroad.

The Koopman et al. (2014) breakdown applies only to aggregate exports, whereas investigating the value-added content of trade at the bilateral level and/or sectoral level is crucial in many cases. Moreover, some of the components of the Koopman et al. (2014) breakdown are imprecisely defined. Here below, we report a sink-based decomposition of bilateral trade flows that is consistent with the aggregate breakdown by Koopman et al. (2014). This allows to point out the differences as compared to the correspondent decomposition presented in Section 3.2.2, and in Appendix E for the foreign value-added.

Sink-based breakdown of bilateral exports

A full sink-based decomposition of bilateral exports from country s to country r can be expressed by the following accounting relationship:

$$\begin{aligned}
\mathbf{u}_N \mathbf{E}_{sr} = {} & \overbrace{\mathbf{V}_s \mathbf{B}_{ss} \mathbf{Y}_{sr}}^{\mathbf{1}} \\
& + \mathbf{V}_s \mathbf{B}_{ss} \mathbf{A}_{sr} (\mathbf{I} - \mathbf{A}_{rr})^{-1} \Bigg[\overbrace{\mathbf{Y}_{rr}}^{\mathbf{2a}} + \overbrace{\sum_{j \neq r}^{G} \mathbf{A}_{rj} \mathbf{B}^{\not s}_{jr} \mathbf{Y}_{rr}}^{\mathbf{2b}} + \overbrace{\sum_{j \neq r}^{G} \mathbf{A}_{rj} \sum_{k \neq s,r}^{G} \mathbf{B}^{\not s}_{jk} \mathbf{Y}_{kk}}^{\mathbf{2c}} \Bigg] \\
& + \mathbf{V}_s \mathbf{B}_{ss} \mathbf{A}_{sr} (\mathbf{I} - \mathbf{A}_{rr})^{-1} \Bigg[\overbrace{\sum_{j \neq r,s}^{G} \mathbf{Y}_{rj}}^{\mathbf{3a}} + \overbrace{\sum_{j \neq r}^{G} \mathbf{A}_{rj} \sum_{l \neq s,r}^{G} \mathbf{B}^{\not s}_{jr} \mathbf{Y}_{rl}}^{\mathbf{3b}}
\end{aligned}$$

$$
\begin{aligned}
& + \sum_{j \neq r}^{G} \mathbf{A}_{rj} \overset{\mathbf{3c}}{\sum_{k \neq s,r}^{G}} \mathbf{B}^{\not s}_{jk} \mathbf{Y}_{kr} + \sum_{j \neq r}^{G} \mathbf{A}_{rj} \overset{\mathbf{3d}}{\sum_{k \neq s,r,l}^{G} \sum_{l \neq s,r}^{G}} \mathbf{B}^{\not s}_{jk} \mathbf{Y}_{kl} \Bigg] \\
& + \mathbf{V}_s \mathbf{B}_{ss} \mathbf{A}_{sr} (\mathbf{I} - \mathbf{A}_{rr})^{-1} \Bigg[\overset{\mathbf{4a}}{\mathbf{Y}_{rs}} + \overset{\mathbf{4b}}{\sum_{j \neq r}^{G} \mathbf{A}_{rj} \mathbf{B}^{\not s}_{jr} \mathbf{Y}_{rs}} + \sum_{j \neq r}^{G} \mathbf{A}_{rj} \overset{\mathbf{4c}}{\sum_{k \neq s,r}^{G}} \mathbf{B}^{\not s}_{jk} \mathbf{Y}_{ks} \Bigg] \\
& + \overset{\mathbf{5}}{\mathbf{V}_s \mathbf{B}_{ss} \mathbf{A}_{sr} (\mathbf{I} - \mathbf{A}_{rr})^{-1} \sum_{j \neq r}^{G} \mathbf{A}_{rj} \mathbf{B}^{\not s}_{js} \mathbf{Y}_{ss}} \\
& + \overset{\mathbf{6}}{\mathbf{V}_s \mathbf{B}_{ss} \mathbf{A}_{sr} (\mathbf{I} - \mathbf{A}_{rr})^{-1} \sum_{j \neq r}^{G} \mathbf{A}_{rj} \mathbf{B}^{\not s}_{js} \mathbf{E}_{s*}} \\
& + \overset{\mathbf{7}}{\sum_{t \neq s}^{G} \mathbf{V}_t \mathbf{B}_{ts} \mathbf{Y}_{sr}} + \overset{\mathbf{8}}{\sum_{t \neq s}^{G} \mathbf{V}_t \mathbf{B}_{ts} \mathbf{A}_{sr} (\mathbf{I} - \mathbf{A}_{rr})^{-1} \mathbf{Y}_{rr}} \\
& + \mathbf{V}_r \mathbf{B}_{rs} \mathbf{A}_{sr} (\mathbf{I} - \mathbf{A}_{rr})^{-1} \Bigg[\overset{\mathbf{9a}}{\sum_{j \neq r}^{G} \mathbf{Y}_{rj}} + \overset{\mathbf{9b}}{\sum_{j \neq r}^{G} \mathbf{A}_{rj} (\mathbf{I} - \mathbf{A}_{jj})^{-1} \mathbf{Y}_{jj}} \Bigg] \\
& + \overset{\mathbf{9c}}{\sum_{t \neq s,r}^{G} \mathbf{V}_t \mathbf{B}_{ts} \mathbf{A}_{sr} (\mathbf{I} - \mathbf{A}_{rr})^{-1} \mathbf{E}_{r*}} \\
& + \overset{\mathbf{9d}}{\mathbf{V}_r \mathbf{B}_{rs} \mathbf{A}_{sr} (\mathbf{I} - \mathbf{A}_{rr})^{-1} \sum_{j \neq r}^{G} \mathbf{A}_{rj} (\mathbf{I} - \mathbf{A}_{jj})^{-1} \mathbf{E}_{j*}}
\end{aligned}
\tag{D.2}
$$

We can define the items that form the bilateral decomposition of gross exports as follows:

- **1** domestic value-added (VA) in direct final good exports;
- **2a** domestic VA in intermediate exports absorbed by direct importers as local final goods;
- **2b** domestic VA in intermediate exports absorbed by direct importers as local final goods only after additional processing stages abroad;
- **2c** domestic VA in intermediate exports absorbed by third countries as local final goods;
- **3a** domestic VA in intermediate exports absorbed by third countries as final goods from direct bilateral importers;
- **3b** domestic VA in intermediate exports absorbed by third countries as final goods from direct bilateral importers only after further processing stages abroad;
- **3c** domestic VA in intermediate exports absorbed by direct importers as final goods from third countries;
- **3d** domestic VA in intermediate exports absorbed by third countries as final goods from other third countries;

4a domestic VA in intermediate exports absorbed at home as final goods of the bilateral importers;

4b domestic VA in intermediate exports absorbed at home as final goods of the bilateral importers after additional processing stages abroad;

4c domestic VA in intermediate exports absorbed at home as final goods of a third country;

5 domestic VA in intermediate exports absorbed at home as domestic final goods;

6 double-counted intermediate exports originally produced at home;

7 foreign VA in exports of final goods;

8 foreign VA in exports of intermediate goods directly absorbed by the importing country r;

9a and 9b foreign VA in exports of intermediate goods re-exported by r directly to the country of final absorption.

9c and 9d double-counted intermediate exports originally produced abroad.

The enumeration of the items recalls the original Koopman et al. (2014) components, which can be obtained as a simple summation over the importing countries r of the corresponding items in our bilateral decomposition (e.g. the second term in KWW is equal to the sum across the r destinations, $\mathbf{E}_{s*} = \sum_{r \neq s}^{G} \mathbf{E}_{sr}$, of **2a**+**2b**+**2c**).

We can then provide formal proof of this equivalence for each item in equation (D.2).

For items **1**, **7** and **8** the original KWW components can be obtained as a simple sum over the importing countries r of the corresponding items in our bilateral decomposition.

As already mentioned, despite being labeled by KWW as foreign double counted, item **9** in the original Koopman et al. (2014) decomposition (D.1) includes both foreign value-added—which corresponds to terms **9a** and **9b** in equation (D.2)—and foreign double counted components—i.e. terms **9c** and **9d**. Then, item **9** of the KWW decomposition can be obtained by aggregating terms **9a**–**9d** in (D.2), and summing across all the importing partners r. To show this, we can first split term **9** of equation (D.1) into the part originally produced by the importing country r and that produced elsewhere:

$$\sum_{t \neq s}^{G} \sum_{r \neq s}^{G} \mathbf{V}_t \mathbf{B}_{ts} \mathbf{A}_{sr} (\mathbf{I} - \mathbf{A}_{rr})^{-1} \mathbf{E}_{r*} = \sum_{t \neq s,r}^{G} \sum_{r \neq s}^{G} \mathbf{V}_t \mathbf{B}_{ts} \mathbf{A}_{sr} (\mathbf{I} - \mathbf{A}_{rr})^{-1} \mathbf{E}_{r*} + \sum_{r \neq s}^{G} \mathbf{V}_r \mathbf{B}_{rs} \mathbf{A}_{sr} (\mathbf{I} - \mathbf{A}_{rr})^{-1} \mathbf{E}_{r*}. \tag{D.3}$$

The first item on the right-hand side of equation (D.3) corresponds to the sum across bilateral partners of the item **9c**. Items **9a**, **9b** and **9d** can be easily identified starting from the second item on the right-hand side of equation (D.3) and expressing the exports from countries r ($\mathbf{E}_{r*}$) according to equations (5), (8) and (9):

$$\sum_{t\neq s}^{G}\sum_{r\neq s}^{G}\mathbf{V}_t\mathbf{B}_{ts}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{E}_{r*} = \sum_{t\neq s,r}^{G}\sum_{r\neq s}^{G}\mathbf{V}_t\mathbf{B}_{ts}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{E}_{r*}$$
$$+ \sum_{r\neq s}^{G}\mathbf{V}_r\mathbf{B}_{rs}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{A}_{rj}(\mathbf{I}-\mathbf{A}_{jj})^{-1}\mathbf{Y}_{jj}$$
$$+ \sum_{r\neq s}^{G}\mathbf{V}_r\mathbf{B}_{rs}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{A}_{rj}(\mathbf{I}-\mathbf{A}_{jj})^{-1}\mathbf{E}_{j*}$$
$$+ \sum_{r\neq s}^{G}\mathbf{V}_r\mathbf{B}_{rs}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{Y}_{rj}. \quad \text{(D.4)}$$

For the remaining components, a few more steps are needed to prove the equivalence between the two expressions.

Plugging (B.9) and (B.10) into (B.8), we obtain the following expression for $\mathbf{B}_{sr}$:

$$\mathbf{B}_{sr} = \mathbf{B}_{ss}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1} + \mathbf{B}_{ss}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}\mathbf{A}_{rj}\mathbf{B}^{\not s}_{jr}$$
$$+ \mathbf{B}_{ss}\sum_{t\neq s,r}\mathbf{A}_{st}(\mathbf{I}-\mathbf{A}_{tt})^{-1}\sum_{j\neq t}^{G}\mathbf{A}_{tj}\mathbf{B}^{\not s}_{jr}. \quad \text{(D.5)}$$

Finally, we can sum across the $G-1$ foreign countries (i.e. $\sum_{r\neq s}^{G}$) to show that the remaining items in the accounting of bilateral trade flows in equation (D.2) can be mapped into the corresponding components of the original KWW decomposition of aggregate exports. For instance, pre-multiplying by matrix $\mathbf{V}_s$, post-multiplying by $\mathbf{Y}_{rr}$ and summing across r both sides of equation (D.5) we exactly retrieve the second component of the KWW decomposition:

$$\begin{aligned}\mathbf{V}_s\sum_{r\neq s}\mathbf{B}_{sr}\mathbf{Y}_{rr} &= \mathbf{V}_s\sum_{r\neq s}\mathbf{B}_{ss}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{Y}_{rr}\\ &+ \mathbf{V}_s\sum_{r\neq s}\mathbf{B}_{ss}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}\mathbf{A}_{rj}\mathbf{B}^{\not s}_{jr}\mathbf{Y}_{rr}\\ &+ \mathbf{V}_s\sum_{r\neq s}\mathbf{B}_{ss}\sum_{t\neq s,r}\mathbf{A}_{st}(\mathbf{I}-\mathbf{A}_{tt})^{-1}\sum_{j\neq t}^{G}\mathbf{A}_{tj}\mathbf{B}^{\not s}_{jr}\mathbf{Y}_{rr}\end{aligned} \quad \text{(D.6)}$$

where the left-hand side of equation (D.6) corresponds to the sum across all direct importers (r) of the components **2a**, **2b** and **2c** in equation (D.2):

$$\begin{aligned}\sum_{r\neq s}(\mathbf{2a}+\mathbf{2b}+\mathbf{2c}) &= \mathbf{V}_s\sum_{r\neq s}\mathbf{B}_{ss}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{Y}_{rr}\\ &+ \mathbf{V}_s\sum_{r\neq s}\mathbf{B}_{ss}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{A}_{rj}\mathbf{B}^{\not s}_{jr}\mathbf{Y}_{rr}\\ &+ \mathbf{V}_s\sum_{r\neq s}\mathbf{B}_{ss}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{A}_{rj}\sum_{k\neq s,r}^{G}\mathbf{B}^{\not s}_{jk}\mathbf{Y}_{kk}\end{aligned} \quad \text{(D.7)}$$

The first two terms on the left-hand side are clearly identical, and the equivalence between the last items is readily verified by replacing the subscript k with r and the subscript r with t in the last term of equation (D.7). However, it should be noticed that for this last term the single addends in the summation across the r foreign countries differ between the two equations. This is because this portion of domestic value-added produced in s for final use in r gets to the final destination markets by passing through one or more third countries; that is, it is not part of the bilateral exports from s to r.

Starting from the definition of the $\mathbf{B}_{rs}$ matrix in equation (D.5) and following the same procedure employed for the second item of the KWW decomposition, it is easy to prove that the third and fourth components too can be obtained as the sum of the corresponding items in our bilateral decomposition across all the destinations.

Finally, we use a slightly different procedure to show that also the fifth and sixth terms in the KWW main accounting relationship are exactly mapped within the bilateral exports. We start by singling out the block matrix $\mathbf{B}_{ss}$ from the principal diagonal of the $\mathbf{B}$ matrix. According to equation (B.6) this matrix is equal to:

$$\mathbf{B}_{ss} = \mathbf{B}^{\not s}_{ss} + \mathbf{B}_{ss} \sum_{r \neq s} \mathbf{A}_{sr} \mathbf{B}^{\not s}_{rs} \tag{D.8}$$

We can then apply to the $\mathbf{B}^{\not s}$ the property of the block diagonal elements of the $\mathbf{B}$ matrix illustrated in (B.1):

$$\mathbf{B}^{\not s}_{ss} = (\mathbf{I} - \mathbf{A}^{\not s}_{ss})^{-1} + \sum_{t \neq s} \mathbf{B}^{\not s}_{st} \mathbf{A}_{ts} (\mathbf{I} - \mathbf{A}^{\not s}_{ss})^{-1} = (\mathbf{I} - \mathbf{A}_{ss})^{-1} \tag{D.9}$$

where the last equality follows from the fact that, by construction, $\mathbf{B}^{\not s}_{st}$ is equal to $\mathbf{0}$ for each $t \neq s$. Therefore (D.8) can be rewritten as follows:

$$\mathbf{B}_{ss} = (\mathbf{I} - \mathbf{A}_{ss})^{-1} + \mathbf{B}_{ss} \sum_{r \neq s} \mathbf{A}_{sr} \mathbf{B}^{\not s}_{rs} \tag{D.10}$$

Then, applying the same property of the block diagonal elements of the $\mathbf{B}$ matrix to the left-hand side of (D.10) and rearranging we obtain:

$$\sum_{r \neq s} \mathbf{B}_{sr} \mathbf{A}_{rs} (\mathbf{I} - \mathbf{A}_{ss})^{-1} = \sum_{r \neq s} \mathbf{B}_{ss} \mathbf{A}_{sr} \mathbf{B}^{\not s}_{rs} \tag{D.11}$$

Finally, using the property presented in (B.2) to the $\mathbf{B}^{\not s}_{rs}$ matrix we get:

$$\sum_{r \neq s} \mathbf{B}_{sr} \mathbf{A}_{rs} (\mathbf{I} - \mathbf{A}_{ss})^{-1} = \sum_{r \neq s} \mathbf{B}_{ss} \mathbf{A}_{sr} (\mathbf{I} - \mathbf{A}_{rr})^{-1} \sum_{j \neq r} \mathbf{A}_{rj} \mathbf{B}^{\not s}_{js} \tag{D.12}$$

Now it is straightforward to see that the fifth and sixth terms in the KWW decomposition are simply the sum of the same terms in equation (D.2) across all the bilateral destinations.

E Appendix: World perspective for the breakdown of foreign content of exports

As already mentioned, the breakdowns proposed in Section 3 use a notion of 'foreign double counted' (FDC) that shares the same rationale as the 'domestic double counted': we only include the items that cross the same (i.e. the exporter's) border more than once in the FDC. In other words, we use an 'exporting country-level perspective' for both the accounting of domestic and foreign content of exports.

Other methodologies, which also use an exporting country-level perspective for the domestic component, take a different approach for the foreign content of exports:[20] a certain item is considered as value-added only the first (or the last) time it crosses a foreign border, whereas all the other times it crosses any foreign border it is classified as double counted. We can label this approach a 'world-level perspective', since all trade flows—not only the exports of a single country—are considered in order to single out the items that are exported multiple times. In other words, with the 'exporting country-level perspective' a certain item is accounted for as FVA only once in the total exports of a country, whereas the 'world-level perspective' requires it to be accounted for as FVA only once in total world exports.

In order to better appreciate the difference between the two approaches, it is useful to re-express the decompositions of the foreign content of export according to a 'world-level perspective'. The distinction between the source- and sink-based approaches also applies in this case. The source-based approach requires a certain item to be recorded as 'foreign value-added' the first time it is re-exported by a country other than the country of origin and it is the one followed by Miroudot and Ye (2020). Here we also consider the country of final completion and the market of final destination:

$$\begin{aligned}\mathbf{FVA}source_{sr}^{WP} = \sum_{t\neq s}^{G} \mathbf{V}_t(\mathbf{I}-\mathbf{A}_{tt})^{-1}\mathbf{A}_{ts}(\mathbf{I}-\mathbf{A}_{ss})^{-1}\Bigg[\mathbf{Y}_{sr} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{Y}_{rr} \\ + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{Y}_{rj} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{A}_{rj}\sum_{k}^{G}\sum_{l}^{G}\mathbf{B}_{jk}\mathbf{Y}_{kl}\Bigg]\end{aligned} \tag{E.1}$$

$$\mathbf{FDC}source_{sr}^{WP} = \sum_{t\neq s}^{G} \mathbf{V}_t(\mathbf{I}-\mathbf{A}_{tt})^{-1}\left[\sum_{j\neq t,s}^{G}\mathbf{A}_{tj}\mathbf{B}_{js}\mathbf{E}_{sr} + \mathbf{A}_{ts}(\mathbf{I}-\mathbf{A}_{ss})\sum_{t\neq s}^{G}\mathbf{A}_{st}\mathbf{B}_{ts}\mathbf{E}_{sr}\right]. \tag{E.2}$$

The foreign content of exports can also be decomposed using a sink-based approach, while maintaining a 'world-level perspective'. In this case, a given item is accounted for as 'foreign value-added' the last time it is exported by a country that is not the country of origin. This logic is also followed by Koopman et al. (2014), however, this part of their decomposition is affected by some specific drawbacks discussed in Appendix D.

[20] The following contributions fit into this category: Koopman et al. (2014), Wang et al. (2013), Nagengast and Stehrer (2016), Miroudot and Ye (2020).

The value-added and double-counted components of the foreign content of exports in a world-perspective/sink-based approach can be expressed as follows:

$$\mathbf{FVA}sink_{sr}^{WP} = \sum_{t\neq s}^{G} \mathbf{V}_t \mathbf{B}_{ts} \mathbf{Y}_{sr} + \sum_{t\neq s}^{G} \mathbf{V}_t \mathbf{B}_{ts} \mathbf{A}_{sr} (\mathbf{I} - \mathbf{A}_{rr})^{-1} \mathbf{Y}_{rr} + \mathbf{V}_r \mathbf{B}_{rs} \mathbf{A}_{sr} (\mathbf{I} - \mathbf{A}_{rr})^{-1} \left[\sum_{j\neq r}^{G} \mathbf{Y}_{rj} + \sum_{j\neq r}^{G} \mathbf{A}_{rj} (\mathbf{I} - \mathbf{A}_{jj})^{-1} \mathbf{Y}_{jj} \right] \quad \text{(E.3)}$$

$$\mathbf{FDC}sink_{sr}^{WP} = \sum_{t\neq s,r}^{G} \mathbf{V}_t \mathbf{B}_{ts} \mathbf{A}_{sr} (\mathbf{I} - \mathbf{A}_{rr})^{-1} \mathbf{E}_{r*} + \mathbf{V}_r \mathbf{B}_{rs} \mathbf{A}_{sr} (\mathbf{I} - \mathbf{A}_{rr})^{-1} \sum_{j\neq r}^{G} \mathbf{A}_{rj} (\mathbf{I} - \mathbf{A}_{jj})^{-1} \mathbf{E}_{j*}. \quad \text{(E.4)}$$

If we consider an exporter s and we sum across the bilateral destinations r the FVA and FDC indicators in equations (E.1)–(E.4) we obtain different values for the FVA and the FDC. Indeed, whenever the first re-exporting country does not coincide with the last, the source-based indicators of foreign content and the sink-based ones differ when computed at the level of the exporting country. However, both approaches record a given item as FVA only once in world trade flows. This means that when aggregating at the world level the components in equations (E.1)–(E.4) (i.e. summing across all the exporters and the importers) we obtain exactly the same measures of FVA and FDC.

Conversely the source and sink decompositions of the foreign content of exports based on an 'exporting country-level perspective' are completely consistent with each other when we sum across all destination markets (i.e. considering equations (G.3) and (G.9), $\sum_{r\neq s}^{G} \mathbf{FVA}sink_{sr} = \sum_{r\neq s}^{G} \mathbf{FVA}source_{sr}$). Thus, the exporting country-level approach leads to a unique measure of FVA (and FDC) at the country level.

The decompositions based on a 'world-level perspective' can be used to address important questions regarding the breakdown of total world trade. For instance, we can measure the share of world GDP that enters the exports of some other country. However, these measures seem particularly unsatisfactory when analyzing the exports of a given country. Indeed, in the breakdown of a country's exports, this distinction between FVA and FDC turns out to be totally arbitrary. Consider the following example: China imports intermediates directly from Germany and indirectly from France; according to a world-perspective source-based approach, the German VA is considered as FVA in Chinese exports, whereas the French VA is classified as FDC, even if the two components contribute in a very similar way to the value embedded in Chinese exports.[21]

From the perspective of the exporting country, we are usually interested in measuring the share of exports that can be traced back to the domestic and foreign GDP, irrespective of the number of upstream or downstream production stages that separate the exporter from the country of origin and/or the market of final destination. Indeed the 'foreign value-added' indicators based on an 'exporting country-level perspective' address

[21] Likewise the sink-based classification produces arbitrary allocations too.

the following questions: which part of a country's exports can be traced back to another country's GDP? Alternatively, what is the portion of a country's GDP that is embedded into the exports of a certain other country or of a certain bilateral flow? These questions might be particularly relevant from a policy point of view, for instance when discussing the impact of a trade policy on third-country production.

In principle, the FVA calculated according to the world-level perspective can be used to address the following question: 'what part of country s's GDP enters other countries' exports?' In particular, it can be computed through equation (E.1) (or in (E.3)) by tracking a specific country of origin of the value-added t in the exports of all the other exporting countries $s \neq t$ toward all the bilateral partners r. However, the same question can be addressed directly in the exporting country-level source-based framework by subtracting the component of direct absorption from the total DVA of the exporting country itself (i.e. $\sum_{r \neq s}^{G} \mathbf{DVA}source_{sr} - \sum_{r \neq s}^{G} \mathbf{DAVAX}_{sr}$, see equations (G.1) and (10)).

Finally, it is worth noting that at world level all the FVA components are also recorded as DVA in the flows of other countries, meaning that they are already double counted GDP in exports. It is thus questionable to argue a certain item should be recorded as FVA only once in world trade flows in order to avoid double counting of the same production in trade.

F Appendix: Differences between exporter perceptive indicators and bilateral perceptive ones in bilateral trade decomposition

In this Appendix we show how the terms of the decomposition based on a bilateral perspective are related to the terms in the bilateral decompositions based on an exporting country-level perspective presented in Section 4. First, since the breakdown in equation (16) hinges on a less restrictive notion of 'double counting' as compared to the exporting country-level perspective, both the sink and source measures of domestic and foreign value-added are part of the corresponding terms derived in the bilateral perspective decomposition (i.e. $\mathbf{DVA}source_{sr} \subset \mathbf{DVA}^*_{sr}$, $\mathbf{FVA}source_{sr} \subset \mathbf{FVA}^*_{sr}$; $\mathbf{DVA}sink_{sr} \subset \mathbf{DVA}^*_{sr}$, $\mathbf{FVA}sink_{sr} \subset \mathbf{FVA}^*_{sr}$). We can then break down the double counted terms of the decompositions based on an exporting country level perspective in order to single out the subcomponents that are classified differently according to the perspective adopted. For instance, if we consider the domestic double-counted component ($\mathbf{DDC}_{sr}$), we can re-express the original indicators as follows:

$$\mathbf{DDC}source_{sr} = \underbrace{\mathbf{V}_s \mathbf{B}^{\not{s}}_{ss} \sum_{j \neq s,r} \mathbf{A}_{sj} \mathbf{B}^{\not{sr}}_{js} \mathbf{E}_{sr}}_{\mathbf{DVA}^{*(\mathbf{DDC}source)}_{sr}} + \mathbf{DDC}^*_{sr} \tag{F.1}$$

$$\mathbf{DDC}sink_{sr} = \underbrace{\mathbf{V}_s \mathbf{B}_{ss} \mathbf{A}_{sr} \mathbf{B}^{\not{s}}_{rs} \sum_{i \neq s,r}^{G} \left[\mathbf{Y}_{si} + \mathbf{A}_{si} \mathbf{B}^{\not{sr}}_{is} \sum_{l \neq r}^{G} \mathbf{Y}_{sl} + \mathbf{A}_{si} \sum_{k \neq s}^{G} \sum_{l}^{G} \mathbf{B}^{\not{sr}}_{ik} \mathbf{Y}_{kl} \right]}_{\mathbf{DVA}^{*(\mathbf{DDC}sink)}_{sr}} + \mathbf{DDC}^*_{sr}, \tag{F.2}$$

where $\mathbf{DVA}^{*(\mathbf{DDC}source)}_{sr}$ and $\mathbf{DVA}^{*(\mathbf{DDC}sink)}_{sr}$ are the components that are classified as 'double counted' respectively in the source and sink decompositions based on an exporting country-level perspective, but are classified as 'domestic value-added' in a purely bilateral perspective. How these components should be considered depends on the specific economic issue under investigation. It is also worth noting that the following relationship holds true:

$$\mathbf{DVA}^*_{sr} = \mathbf{DVA}source_{sr} + \mathbf{DVA}^{*(\mathbf{DDC}source)}_{sr} = \mathbf{DVA}sink_{sr} + \mathbf{DVA}^{*(\mathbf{DDC}sink)}_{sr}. \tag{F.3}$$

The derivation of equations (F.1)–(F.3) can also be applied to single out the differences between the foreign value added in a bilateral-level perspective ($\mathbf{FVA}^*_{sr}$) and those defined in an exporting country-level perspective ($\mathbf{FVA}source_{sr}$ and $\mathbf{FVA}sink_{sr}$).

G Appendix: Complete decompositions of bilateral exports according to the exporting country perspective

In this Appendix we present the complete formulas for the source- and sink-based decompoistions of bilateral exports presented in Section 3.2.1 and 3.2.2 considering both the country of origin and the market of final destination. In line with the main scheme proposed by Koopman et al. (2014) we show the following components: domestic value-added (DVA), domestic double-counted (DDC), foreign value-added (FVA), foreign double-counted (DDC). We also show the split of the domestic value-added in the part absorbed abroad (value-added exports, VAX) and the part absorbed by the exporting country itself (reflection, REF).

Source-based breakdown of bilateral exports

$$\mathbf{DVA}source_{sr} = \mathbf{V}_s(\mathbf{I}-\mathbf{A}_{ss})^{-1}\Bigg[\mathbf{Y}_{sr} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{Y}_{rr} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{Y}_{rj} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{A}_{rj}\sum_{k}^{G}\sum_{l}^{G}\mathbf{B}_{jk}\mathbf{Y}_{kl}\Bigg], \tag{G.1}$$

$$\mathbf{DDC}source_{sr} = \mathbf{V}_s(\mathbf{I}-\mathbf{A}_{ss})^{-1}\sum_{j\neq s}\mathbf{A}_{sj}\mathbf{B}_{js}\mathbf{E}_{sr}, \tag{G.2}$$

$$\mathbf{FVA}source_{sr} = \sum_{t\neq s}^{G}\mathbf{V}_t\mathbf{B}_{ts}^{\not s}\Bigg[\mathbf{Y}_{sr} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{Y}_{rr} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{Y}_{rj} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{A}_{rj}\sum_{k}^{G}\sum_{l}^{G}\mathbf{B}_{jk}\mathbf{Y}_{kl}\Bigg], \tag{G.3}$$

$$\mathbf{FDC}source_{sr} = \sum_{t\neq s}^{G}\mathbf{V}_t\mathbf{B}_{ts}^{\not s}\sum_{j\neq s}\mathbf{A}_{sj}\mathbf{B}_{js}\mathbf{E}_{sr}, \tag{G.4}$$

$$\mathbf{REF}source_{sr} = \mathbf{V}_s(\mathbf{I}-\mathbf{A}_{ss})^{-1}\Bigg[\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{Y}_{rs} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{A}_{rj}\sum_{k}^{G}\mathbf{B}_{jk}\mathbf{Y}_{ks}\Bigg] \tag{G.5}$$

$$\mathbf{VAX}source_{sr} = \mathbf{V}_s(\mathbf{I}-\mathbf{A}_{ss})^{-1}\Bigg[\mathbf{Y}_{sr} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{Y}_{rr} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r,s}^{G}\mathbf{Y}_{rj} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{A}_{rj}\sum_{k}^{G}\sum_{l\neq s}^{G}\mathbf{B}_{jk}\mathbf{Y}_{kl}\Bigg]. \tag{G.6}$$

Sink-based breakdown of bilateral exports

$$\mathbf{DVA}sink_{sr} = \mathbf{V}_s\mathbf{B}_{ss}\left[\mathbf{Y}_{sr} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{Y}_{rr} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{Y}_{rj}\right. \\ \left. + \ \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{A}_{rj}\left(\sum_{k\neq s}^{G}\sum_{l}^{G}\mathbf{B}_{jk}^{\not s}\mathbf{Y}_{kl} + \mathbf{B}_{js}^{\not s}\mathbf{Y}_{ss}\right)\right] \tag{G.7}$$

$$\mathbf{DDC}sink_{sr} = \mathbf{V}_s\mathbf{B}_{ss}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{A}_{rj}\mathbf{B}_{js}^{\not s}\mathbf{E}_{s*} \tag{G.8}$$

$$\mathbf{FVA}sink_{sr} = \sum_{t\neq s}^{G}\mathbf{V}_t\mathbf{B}_{ts}\left[\mathbf{Y}_{sr} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{Y}_{rr} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{Y}_{rj}\right. \\ \left. + \ \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{A}_{rj}\left(\sum_{k\neq s}^{G}\sum_{l}^{G}\mathbf{B}_{jk}^{\not s}\mathbf{Y}_{kl} + \mathbf{B}_{js}^{\not s}\mathbf{Y}_{ss}\right)\right] \tag{G.9}$$

$$\mathbf{FDC}sink_{sr} = \sum_{t\neq s}^{G}\mathbf{V}_t\mathbf{B}_{ts}\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{A}_{rj}\mathbf{B}_{js}^{\not s}\mathbf{E}_{s*}. \tag{G.10}$$

$$\mathbf{REF}sink_{sr} = \mathbf{V}_s\mathbf{B}_{ss}\left[\mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{Y}_{rs}\right. \\ \left. + \ \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{A}_{rj}\left(\sum_{k\neq s}^{G}\sum_{l}^{G}\mathbf{B}_{jk}^{\not s}\mathbf{Y}_{ks} + \mathbf{B}_{js}^{\not s}\mathbf{Y}_{ss}\right)\right] \tag{G.11}$$

$$\mathbf{VAX}sink_{sr} = \mathbf{V}_s\mathbf{B}_{ss}\left[\mathbf{Y}_{sr} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\mathbf{Y}_{rr} + \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r,s}^{G}\mathbf{Y}_{rj}\right. \\ \left. + \ \mathbf{A}_{sr}(\mathbf{I}-\mathbf{A}_{rr})^{-1}\sum_{j\neq r}^{G}\mathbf{A}_{rj}\sum_{k\neq s}^{G}\sum_{l\neq s}^{G}\mathbf{B}_{jk}^{\not s}\mathbf{Y}_{kl}\right]. \tag{G.12}$$